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Three essays in macroeconomics

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Three essays in macroeconomics

by

Jiaoting Shi

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Rajesh Singh, Major Professor
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Zhijun Wu

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

Ames, Iowa

2019

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DEDICATION

I would like to dedicate this dissertation to my mother Aiying and to my father Daxue without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance and financial assistance during the writing of this work.

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ABSTRACT

In this dissertation, I offer three essays on macroeconomics with particular attention to the models of search and matching labor and international financial crisis. In plain words, each essay can be interpreted as a real-world narrative, with the proper key word of *overborrowing*, *oversaving*, and *over job-posting*, respectively.

In "Unemployment and Sovereign Risk", we introduce a search and matching labor framework into an otherwise standard sovereign debt model. The interactions between external debt dynamics, domestic labor market outcomes, and time-consistent fiscal policies are analyzed. The incorporation of the frictional labor market is shown to improve the models ability to generate empirically realistic debt level and default frequency. The quantitative impact of high unemployment benefit on deterring vacancy creation not only outweighs its consumption smoothing effect, but also increases vulnerability to a sovereign debt crisis.

Financially integrated economies observe a cross-country credit boom prior to financial recessions and a bust afterwards. In "A Two-country model of Banking Crisis", we present a two-country real business cycle model with banking sector where privately known intermediation efficiency of banks makes them heterogeneous and gives rise to an interbank market. Overaccumulation of assets or low productivity in one country may lead to credit freeze in both financially integrated countries due to the existence of moral hazard and asymmetric information in the interbank market. A "sail together" financial integration may go into a "sink together" interbank credit freeze.

In the homework spirit of exercise, "Efficient Frictions: from credit to labor market" introduces ex-ante heterogeneous skilled workers and endogenous labor market participation into a static credit and labor market search model. In particular, we consider two cases in which workers effectively share the vacancy cost or not, respectively. In the former case, wage contract is settled after loan

contract and workers proportionately share the vacancy cost due to sequential bargaining protocol. In the latter case, workers get paid a constant share of the output due to block bargaining protocol. In both cases, perfect smooth credit is not desired, and a frictional labor market can be welfare improving. Hosios condition is only restored when workers effectively share the vacancy cost.

CHAPTER 1. INTRODUCTION

In this chapter, I will give a brief overview of the three essays on macroeconomics in my dissertation.

In Chapter 2, my job market paper called "Unemployment and Sovereign Risk", we introduce the Diamond-Mortensen-Pissarides (DMP) labor search framework into an otherwise standard Eaton and Gersovitz (1981) sovereign debt model to address two questions: (i) how default risk and fiscal policy affect the labor market outcomes, and (ii) how a tax-financed unemployment benefit scheme affects default decision and sovereign spreads. The answer to these two questions are important in that it may shed light upon the recent debate on whether the government should prefer stimulus over austerity policies during the financial crisis. In particular, our model addresses the trade-off faced with the government of whether to stabilize the labor market or stabilize the whole economy. In this narrative, government mainly provides unemployment benefit to unemployed workers, the proceeds of which are financed through a macro-prudential style capital control tax imposed on the household's debt borrowing from the international financial market. A potential *overborrowing* issue is thus addressed through the tax by making household internalize the pecuniary externality, i.e., the effects of their borrowing decisions on the cost of debt. However, the sovereign government is unable to commit to repay its debt due to limited enforcement and commitment. As a result, when the economy is hit by a large enough negative productivity shock, the government may decide to default and the country will then fall into financial autarky with an explicit output penalty so as to sustain the sovereign borrowing in the first place. There are two main findings in our paper when we calibrate the model to Argentina economy. First, the incorporation of a frictional labor market improves the model's quantitative performance to generate empirically realistic level of debt (around 60% debt-to-GDP ratio) and default frequency (every 40 years). Second, the provision of unemployment benefit deters firms' vacancy creation in the labor

market and makes the country more vulnerable to negative shocks, the effect of which outweighs its consumption smoothing benefit, and makes the country more likely to default. Our model can serve as a framework for analyzing various stabilization and social-insurance policies in the context of sovereign debt crisis.

In Chapter 3, a joint work with Mohammad Hasan called "A Two-country Model of Banking Crises", we extend Boissay et al. (2016) closed-economy version of the banking crisis model to a two-country world economy framework and address international banking crisis with potential contagion and spillover effect through the financially integrated interbank market. In this paper, banks play an otherwise trivial role by absorbing deposits from household and making loans to firms in the retail sector. However, a non-negligible interbank market emerges due to the heterogeneity in banks' intermediation abilities in which relatively more efficient banks borrow funds from those inefficient ones and in turn extend leveraged loans to firms. With moral hazard and incomplete information prevailing in the interbank market, an incentive compatible interbank loan contract gives rise to the so-called "absorbing capacity" of the whole economy. A potential *oversaving* problem from households out of the consumption-smoothing motive following a sequence of positive but small productivity shocks exacerbates overaccumulation of assets. As a result, a negative productivity shock brings the economy's absorbing capacity down below the highly-built pre-crisis asset level and aggravates the interbank market frictions. An increasing number of inefficient banks now find it profitable to borrow in the interbank market and then deviate and repudiate the corresponding loan contract. The interbank credit thus freezes, a financial accelerator mechanism takes in effect, and a financial crisis ensues. In our world economy framework, we consider cases in which the two countries are integrated with mobile deposits only, mobile corporate loans only, or a shared interbank market only. In the last case, one country that experiences a negative shock may drag down the other country together into an international financial crisis through the contagion effect from the connected frozen interbank market. Our quantitative exercise shows that financial crisis occurs less frequently but becomes more severe in the context of a financially integrated world.

In Chapter 4, "Efficient Frictions: From Credit to Labor Market", we introduce ex-ante heterogeneous skilled workers and endogenous labor market participation a la Albrecht et al. (2010) into a single-period version of credit and labor market search model as in Wasmer and Weil (2004). It is shown that the incorporation of a frictional credit market can help restore the Hosios condition¹ and thus improve social welfare. This is because the Hosios condition only addresses the search externality and the hold-up problem in the traditional representative agent search and matching model, but fails to help internalize the so-called "average productivity effect" in a heterogeneous skilled workers setup. We consider two cases in which workers effectively share the vacancy cost or not, respectively. In the first case, wage contract is settled after loan contract due to sequential bargaining protocol and workers have to share the vacancy cost, whereas in the second case, workers get paid a constant share of the output out from the block bargaining protocol. In both cases, perfect smooth credit is not desirable, and a frictional credit market can be welfare improving. However, Hosios condition only holds when workers effectively share the vacancy cost.

In Chapter 5, I address the weaknesses of each paper respectively and discuss some extension and future research work.

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¹See Hosios (1990).

CHAPTER 2. UNEMPLOYMENT AND SOVEREIGN RISK

Modified from a manuscript to be submitted to the Journal of International Economics

Jiaoting Shi

2.1 Abstract

We introduce the Diamond-Mortensen-Pissarides (DMP) labor search framework into an otherwise standard Eaton and Gersovitz (1981) sovereign debt model. The interactions between external debt dynamics, domestic labor market outcomes, and time-consistent fiscal policies are analyzed. The tension between the debt and labor market is realized through the link of the per-period balanced government budget constraint. In a quantitative exercise of calibrating to Argentina economy, we find that the incorporation of the frictional labor market improves the models ability to generate empirically realistic debt level and default frequency. The quantitative impact of high unemployment benefit on deterring vacancy creation not only outweighs its consumption smoothing effect, but also increases vulnerability to a sovereign debt crisis. Our model can serve as a framework for analyzing various stabilization and social-insurance policies in the context of sovereign debt crisis once properly extended to a dynamic labor search environment.

2.2 Introduction

Explaining both labor market fluctuations and sovereign debt crises in a unified framework turns out to be a new challenge in International Macroeconomics. The textbook quantitative sovereign debt models typically assume an endowment economy and thus ignore the interaction between domestic labor market and external debt dynamics. Recent exceptions include Na et al. (2018) and Bianchi et al. (2017). They introduce the notion of downward nominal wage rigidity to these models and study optimal government policies to stabilize unemployment and debt. Yet, they are both full-employment models and thus a thorough anatomy of labor market dynamics is missing. In the spirit of Diamond-Mortensen-Pissarides (DMP), we introduce a frictional labor market featuring search and matching frictions to an otherwise standard sovereign debt model and address (i) how default risk and fiscal policy affect the labor market outcomes, and (ii) how a tax-financed unemployment benefit scheme affects default decision and sovereign spreads.

Exploring labor market dynamics for the analysis of sovereign default is motivated by the concurrent evidence of high unemployment and sovereign spreads in the recent Eurozone debt crisis. Schmitt-Grohe and Uribe (2016) report that the average unemployment rate on the periphery of Europe ¹ reached above 13 percent in 2010. This same episode is featured with the dramatic widening in spreads of yields on 10 year sovereign bonds of Greece, Ireland, Italy, Portugal, Spain (GIIPS) and that of Germany ². What's more, models with search and matching frictions, such as Diamond (1981), Pissarides (1985) and Mortensen and Pissarides (1994), have become the workhorse model for equilibrium unemployment analysis. These models capture important labor market dynamics over the business cycle and are widely used to understand how fiscal policy affects unemployment and output. It is then of significant importance to examine the link between fiscal policy, unemployment, and sovereign default risk implied by these models.

The main findings in the paper are as follows. First, a frictional labor market, in combination with default penalty, improve the model's ability to generate empirically realistic level of debt,

¹It is calculated as the arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.

²See Wright (2014), Figure 1.

default frequency, and the observed volatility of country premium under plausible magnitude of productivity shocks. Second, the quantitative impact of high unemployment benefit on deterring vacancy creation not only outweighs its consumption smoothing effect, the continuous provision of which also increases further borrowing costs and the probability of a sovereign debt crisis. Third, the endogenous unemployment cost of default has contributed to government policy making, but only with a minor effect on the decision to default or not, due to the static labor market setup. Finally, the model cannot simultaneously account for both plausible employment level during normal times and sizable spike-up in unemployment upon default. This finding is robust even if one adopts an alternative calibration strategy which yields nearly 100 percent replacement ratio. As in many other DMP models in the business cycle framework, it reflects the difficulty to get employment to fall during downturns driven by productivity shocks.

Our production economy has four main ingredients. First, a representative household consists of both employed and unemployed workers. This type of aggregation eliminates discrepancy in consumption across household members by providing perfect consumption insurance within the family. Also, the number of unemployed workers in the household represents the total unemployment rate in the economy. Second, debt issuance is conducted by the household while the default decision is up to the central government. As illustrated by Kim and Zhang (2012), decentralized borrowing and centralized default might lead to excessive borrowing by the private sector. This is because individual borrowers are price-takers. They take the country premium as given and thus fail to take into account the effect of their borrowing decisions on the interest rate. To this end, a macroprudential style capital control tax is imposed by the government to help the household internalize the pecuniary externality ³, see, for example, Bianchi (2011), Korinek (2018), and Na et al. (2018). Third, we assume government cannot commit to its future policies and characterize the Markov Perfect Equilibria ⁴ (MPE). The government conduct time-consistent fiscal policies

³Pecuniary externality is not a source of inefficiency if it is merely a general equilibrium effect operated through prices. However, they will induce severe welfare losses if the market is incomplete and there is limited enforcement. For more discussions, see Kim and Zhang (2012) and references therein.

⁴The notion of Markov Perfect Equilibria can be illustrated in the following example. Consider the infinitely-repeated game of prisoners dilemma. Two sub-game perfect equilibria are of interest. One is bad, nasty, and simple in the sense that both players would choose to defect every period. The other one is good, meaning that if both

which can depend only on the current payoff-relevant state variables. That is, it should take into account the effect of today's policy on that of tomorrow's, which in turn shapes the incentives and behavior of today's private sector due to the rational expectation, and thus the feedback from the private sector to today's policy making. In equilibrium, the policy making should be consistent ⁵ over time. Fourth, the baseline model is built with a static labor market. The separation rate of each job-worker pair is one after production and the search and matching process re-start in every period with all household members looking for jobs on the market. As a result, the employment stock is no longer a state variable. This setup facilitates computation and convergence by disentangling the debt and labor market in a way that the dynamics of the latter are purely driven by productivity shocks.

The model is calibrated to match relevant moments of quarterly Argentina data for the 1983-2001 period. I do not have the full dataset of Argentina labor market and thus certain comparisons are missing. The matching function curvature, vacancy cost, and worker's bargaining power are jointly chosen to match the average unemployment rate, hiring cost to wage ratio, and the replacement ratio in the data. Parameters describing shocks, preferences, and the default penalty are calibrated to match the average debt-to-GDP ratio, frequency of default, and output loss. The model also does a good job at replicating a couple of moments that were not targeted in the calibration, such as the positive correlation between the trade balance and the country premium.

In the calibration, the generated cyclical movements of labor market variables seem not to match those observed in the data. The so-called Shimer's puzzle (Shimer, 2005) still prevails in the baseline model even if one adopts the calibration strategy proposed by Hagedorn and Manovskii (2008). This is mainly due to the slightly different interpretation of unemployment in the context of a static labor search setup. In the model, the unemployed workers are not those who are actively

players are patient enough, by folk theorem, a triggering strategy gives that both would choose to cooperate until at least one of them deviates, then both would choose to defect thereafter, a tit-for-tat spirit. In this example, the first one is a Markov Perfect Equilibrium while the second one is not.

⁵The source of time-inconsistency does not come from the preferences per se, but from the government budget constraint. See, for example, Klein et al. (2008).

searching for jobs but those who couldn't get successful matches with firm's vacancies. The nature of the residually determined unemployment makes it barely move.

With a fixed threshold associated with default penalty, the model can match neither the level nor the volatility of spreads. There are two reasons. First, Na et al. (2018) argues that with risk-neutral foreign lenders, the average country premium should approximately be equal to the average frequency of default adjusted for the probability of being in financial autarky. Next, as pointed out by Chatterjee and Eyigungor (2012), it is important to allow for a quadratic structure of default punishment in generating realistic volatility of sovereign spreads.

The contribution of the paper can be evaluated in three different dimensions. First, it shows that unemployment dynamics can play an important role in shaping the cost of sovereign default. This channel is already highlighted in Balke and Ravn (2016) and Balke (2017) with a slightly different approach and can complement the imported intermediate goods channel explored by Mendoza and Yue (2012). Second, it links the labor market outcomes to debt market dynamics only through the balanced government budget constraint, i.e., the financing need of unemployment benefit is met by imposing the capital control tax and balanced with the residually determined lump-sum transfer. In other words, it abstracts from any intertwined dynamics between the two markets or any spillover effect. This could be a good reference point for evaluating the cost and benefit of maintaining a balanced government budget constraint during the default episode. Third, it studies the effect of high unemployment benefit in a calibrated framework. Our finding echoes that of Krusell et al. (2010) which shows that the depressing impact of higher unemployment benefit on firm entry outweighs its beneficial effect for consumption smoothing in terms of welfare outcomes. Despite several missing targets from calibration, mostly due to the static labor market setup, our model provides a starting point for incorporating the endogenous disaster dynamics recently explored in the baseline DMP framework by Petrosky-Nadeau et al. (2018).

This paper is related to two large strands of literature. The first strand is the sovereign debt framework⁶ pioneered by Eaton and Gersovitz (1981) and formalized and developed by Arellano

⁶For handbook chapters, see Aguiar and Amador (2014) and Aguiar et al. (2016). For an excellent textbook treatment, see Uribe and Schmitt-Grohe (2017). For recent reviews on the politics and economics aspect of sovereign

(2008) and Aguiar and Gopinath (2006). An important generalization in these quantitative applications is to introduce ex-post direct exogenous output costs of default so as to support ex-ante empirically plausible level of external debt. Indeed, as pointed out by Bulow and Rogoff (1989), the threat of credit exclusion alone (i.e. the reputational mechanism) cannot sustain sovereign debt, as long as government can still save after default. This is also consistent with the negligible role of credit market exclusion for the quantitative performance of these models (Uribe and Schmitt-Groh, 2017). Attempts to endogenize default costs include Mendoza and Yue (2012). They argue that when domestic intermediate goods are imperfect substitutes, the decline in the imported inputs due to credit financing exclusion necessarily leads to efficiency loss. All these models only deal with real debt. Phan (2017) recently shows that reputation alone can sustain nominal sovereign debt when markets are incomplete. Mechanisms other than output loss or reputation considerations to sustain debt borrowing are highlighted by Sandleris (2008). He argues that information revelation is crucial to enforce debt repayment as the repayment/default decision is interpreted as a signal used by the government to communicate information to domestic and foreign agents about the fundamentals of the economy.

The second related strand literature is the labor search and matching framework started from Diamond-Mortensen-Pissarides (DMP). Recent work incorporating DMP models into real business cycle analysis includes Merz (1995), Anfolatto (1996) and Den Haan et al. (2000). They show that the quantitative performance of the real business cycle model can be improved significantly when DMP model is embedded into it. However, Shimer (2005) finds that the standard DMP model cannot generate observed business-cycle-frequency movements in unemployment and job vacancies in response to productivity shocks of a plausible magnitude when the model is calibrated to U.S. data, a.k.a., the Shimer puzzle ⁷. The reason, as Shimer (2005) argues, is that wage is so flexible that most productivity increase are absorbed by higher wages. As a result, the incentive

default, see Hatchondo and Martinez (2010) and Hatchondo et al. (2007), respectively. For empirical evidence, see Tomz and Wright (2013). For discussion on the solution methods in quantitative sovereign debt models, see Hatchondo et al. (2010).

⁷Most early business cycle models cannot get employment to fall enough in downturns driven by productivity shocks. This same problem in the generic business cycle models is inherited by search models, except it's even worse.

for vacancy creation is eliminated and little impact is on unemployment, vacancies, and job-finding rate. Solutions such as introducing wage rigidity, comparably high outside options for workers, and procyclical vacancy costs are proposed ⁸ In our model, the wage determination by Nash bargaining protocol is preserved since it is consistent with the wage flexibility suggested from the micro data.

Our paper is related to Na et al. (2018), Bianchi et al. (2017), Balke and Ravn (2016), and Balke (2017) who also explicitly introduce a labor market into the Eaton and Gersovitz (1981) model. In Na et al. (2018), the government finds it optimal to devalue its nominal exchange rate upon default to lower the real value of wages, thereby reducing involuntary unemployment inflicted by the downward nominal wage rigidity. In this way, the Twin Ds, as emphasized in Reinhart (2002), i.e., the sovereign defaults are accompanied by large devaluations of the nominal exchange rate, is rationalized as an optimal policy outcome. In Bianchi et al. (2017), the option of currency devaluation is not available either because the economy is operating at an fixed exchange rate or equivalently a member of a currency union, and the way to stabilize the labor market and the whole economy is to increase government spending. However, such an expansionary fiscal policy comes at a cost of increasing sovereign spreads and the stabilization effects are shown to be highly non-linear in the severity of the recession. In this sense, their paper joins the recent policy debate of austerity v.s. stimulus for Eurozone crisis management. Other papers along the same line include Arellano and Bai (2016) and Cuadra et al. (2010). All these models assume Walrasian labor markets.

Balke and Ravn (2016) also incorporate the DMP labor search framework into a sovereign debt model to evaluate time-consistent fiscal policy . There are three main differences between theirs and our work. First, they follow the conventional approach of centralized borrowing while we assume that the government retains only the decisions to default and to conduct fiscal policy and let private households borrow . In their setup, borrowing and default decisions are always optimal as long as the government is benevolent. This is because it internalizes that the interest rate faced by the country in international financial markets depends on its net external debt position (Uribe and Schmitt-Groh, 2017). Second, they focus on moral hazard concerns that affect worker's willingness to search

⁸See, for example, Shimer (2004), Hagedorn and Manovskii (2008), and Petrosky-Nadeau and Zhang (2017).

for jobs, as most literature on optimal unemployment insurance (UI) does. On the contrary, Krusell et al. (2010) highlights the trade-off between insurance and job creation in a Bewley type incomplete market model embedded with labor search frictions. Our work complements the latter by using a representative household setup in a sovereign debt model. Third, the unemployment benefit is a policy instrument in their framework while we arbitrarily calibrate it at a fixed level. They show that it is optimal to reduce unemployment benefit during the recession to encourage more active search for jobs so as to support employment. However, this result is neither consistent with the real world policy practice nor the prescriptions offered from the optimal UI literature.

Our work is also related to the emerging literature on optimal unemployment insurance over the business cycle. Recent papers include Nakajima (2012), Landais et al. (2013), Jung and Kuester (2015), Mitman and Rabinovich (2015), and Pei and Xie (2017). Unlike rest of the papers, Nakajima (2012) allows agents to borrow and save. He quantifies the effect of ongoing UI benefit extensions on the unemployment rate using a calibrated structural model that features job search and consumption-saving decisions, skill depreciation, and UI eligibility. Mitman and Rabinovich (2015) characterize optimal cyclical behavior of unemployment insurance and shows that it should rise on impact when the economy is hit by a negative productivity shock but then fall during the recovery. They assume government can commit to future policies while Pei and Xie (2017) relax this assumption and examine the time-consistent policy both qualitatively and quantitatively. Both Landais et al. (2013) and Jung and Kuester (2015) introduce shocks to the worker's bargaining power that are negatively correlated with productivity. Their results, on the contrary, suggest that the optimal variation in unemployment benefits is quantitatively small and short-lived.

The remainder of the paper is organized as follows. Section 2 describes the model and characterizes optimal government policies. Section 3 contains the calibration strategy and quantitative analysis. Section 4 discusses extension. Section 5 concludes.

2.3 The Model

The model is a Diamond-Mortensen-Pissarides (henceforth, DMP) model embedded in an otherwise standard quantitative sovereign debt framework la Eaton and Gersovitz (1981). Time is discrete and extended into an infinite horizon. We consider a small open economy populated by a representative household, a representative firm, and a government. In each period the household can issue a non-state-contingent bond to the risk-neutral lenders in the international financial market.

2.3.1 The Household

The representative household consists of a continuum of workers of measure unity. Workers can be either employed, in which case they earn market wage w_t , or unemployed, and receive unemployment benefit b from the government. In the following static labor market case, we assume b is not a policy instrument but instead a fixed parameter to be calibrated. The household also owns the firm and receives aggregate dividend D_t . In addition, it can borrow and lend in the international financial market by issuing a non-state-contingent bond at price q_t . Here we assume that the household can commit to repay their debt that are subject to an ad hoc natural debt limit. Household members ⁹ pool income together and achieve full consumption insurance within the family. Thus each member consumes the same level of consumption, denoted by C_t . Let N_t denote the number of employed household members. We assume the household are risk averse and are trying to maximize utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$

subject to the budget constraint

$$C_t + d_t = w_t N_t + b(1 - N_t) + (1 - \tau_t) q_t d_{t+1} - T_t + D_t \quad (2.1)$$

⁹We refer interchangeably to workers or household members.

where τ_t is a macro-prudential or capital control tax on debt imposed by the government, and T_t is the lump-sum tax (subsidy if negative). Consumption Euler equation is thus

$$(1 - \tau_t)q_t C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma}$$

The labor market contract is assumed to last for one period only. In other words, at the beginning of each period all household members are unemployed and are searching for jobs in the labor market. Some members find jobs and others do not. At the end of the period, all worker-job pairs separate and the whole search and matching process restart in the next period and so on. The number of jobs is given by

$$N_t = M(1, v_t) \quad (2.2)$$

where v_t are the number of vacancies posted by the firm. The matching function is constant returns to scale, strictly increasing and concave, and satisfies the property: $M(1, v_t) \leq v_t$ due to the uncoordinated nature of the search process. The probability that an unemployed worker finds a job (the job-finding rate) is thus $f_t^u = m_t$ and the probability that an empty vacancy is filled (the job-filling rate) is $f_t^v = m_t/v_t$. Labor market tightness is pinned down by v_t only and we have

$$f_t^u = M(1, v_t) = v_t f(v_t), f_t^v = M(1, v_t)/v_t = f(v_t)$$

note that $f'(v_t) < 0$. That is, a relatively tighter labor market makes it easier for a worker to find a job but harder for a firm to get its vacancy filled. As a result, the number of unemployed workers is given by

$$u_t = 1 - N_t = 1 - v_t f(v_t)$$

2.3.2 The Firm

The representative firm posts vacancy v_t to hire workers and produce output according to a linear form technology. Production is subject to an aggregate productivity shock. The firms profit maximization problem is

$$\max_{v_t} \tilde{z}_t N_t - w_t N_t - v_t k$$

subject to equation (2.2), where \tilde{z}_t is the aggregate productivity and k is the per vacancy cost, respectively. As will be specified later, we adopt the following form of matching function as in Den Haan et al. (2000):

$$M(1, v_t) = \frac{v_t}{(1 + v_t)^{1/\iota}}$$

The choice of the matching technology over the commonly used Cobb-Douglas specification is to guarantee that the job-finding rate and the job-filling rate are always between 0 and 1. Solve the firms problem and we get the number of vacancies

$$v_t = \left[\left(\frac{k}{\tilde{z}_t - w_t} \right)^{-\iota} - 1 \right]^{\frac{1}{\iota}}$$

2.3.3 Nash Bargaining Wages

The firm and the worker bargains over the wage rate w_t according to the Nash bargaining rule. That is, a matched worker and firm choose w_t to maximize the joint product of their individual payoffs from producing minus their "outside options", i.e.,

$$\max_{w_t} (w_t - b)^\eta (z_t - w_t)^{1-\eta}$$

where $\eta \in [0, 1]$ represents the relative bargaining power of the worker. As η approaches to zero, for example, the firm gets all the bargaining power. Simple algebra gives

$$w_t = (1 - \eta)b + \eta \tilde{z}_t$$

that is, the equilibrium market wage rate is a weighted average of the marginal product of labor, \tilde{z}_t , and the workers outside option, b . Intuitively, the higher the workers outside option, or the greater her bargaining power, the higher the wage rate she can get.

2.3.4 Government

The government sets the level of debt tax and decides whether to default or not. Following Na et al. (2018), we assume that in each period the country can be either in good financial standing or bad financial standing. Let I_t be a relevant binary variable and satisfies

$$I_t = \begin{cases} 1, & \text{good financial standing at } t \\ 0, & \text{bad financial standing at } t \end{cases}$$

Then if $I_t = 1$, the country can choose whether to repay the debt or default. If it defaults in period t , it immediately enters into bad financial standing and $I_t = 0$. As standard in the sovereign debt literature, a defaulted country incurs both output loss and reputation cost. Denote \tilde{z}_t the productivity level net of the loss associated with default:

$$\tilde{z}_t = \begin{cases} z_t, & \text{if } I_t = 1 \\ z_t - L(z_t), & \text{otherwise} \end{cases} \quad (2.3)$$

In addition, a defaulted country is excluded from any access to the international financial market for a finite period of time (i.e. the reputation cost). In other words, when $I_t = 0$, the country regains good standing in the next period with a constant and exogenous probability of λ , and maintains its bad financial standing status with probability $1 - \lambda$. Thus, we have

$$(1 - I_t)d_{t+1} = 0 \quad (2.4)$$

Also, when $I_t = 0$, the government simply confiscates any debt payment to the international lenders and rebates the proceeds to the household via either lump-sum transfers or unemployment benefit, or both. Therefore, the governments sequential budget constraint is given by

$$b(1 - N_t) = \tau_t q_t d_{t+1} + (1 - I_t) d_t + T_t \quad (2.5)$$

By a no-arbitrage condition, the equilibrium rate of return of sovereign bonds should be equal to the risk-free world interest rate adjusted for the probability of default, i.e.,

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0$$

2.3.5 Competitive Equilibrium

Combining (1), (3), (4) and (5) along with the firms per period profit of $D_t = \tilde{z}_t N_t - w_t N_t - v_t k$ yields the following good-market-clearing condition

$$C_t = I_t [q_t d_{t+1} - d_t] + [z_t - (1 - I_t) L(z_t)] N_t - v_t k \quad (2.6)$$

Assume that $\ln z_t$ obeys the law of motion,

$$\ln z_t = \rho \ln z_{t-1} + \mu_t$$

where μ_t is an i.i.d. innovation with mean zero and variance σ_μ^2 , and $|\rho| \in [0, 1)$. We are now ready to define a competitive equilibrium.

Definition 2.3.1. *Given processes $\{z_t, T_t, \tau_t, I_t\}$, the initial condition d_0 , and b , a competitive equilibrium consists of stochastic processes $\{C_t, N_t, d_{t+1}, u_t, v_t, \theta_t, w_t, q_t\}$ satisfying*

$$C_t = I_t [q_t d_{t+1} - d_t] + [z_t - (1 - I_t) L(z_t)] N_t - v_t k \quad (2.7)$$

$$(1 - \tau_t) q_t C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma} \quad (2.8)$$

$$v_t = \left[\left(\frac{k}{\tilde{z}_t - w_t} \right)^{-\iota} - 1 \right]^{\frac{1}{\iota}} \quad (2.9)$$

$$N_t = \frac{v_t}{(1 + v_t^\iota)^{1/\iota}} \quad (2.10)$$

$$u_t = 1 - N_t \quad (2.11)$$

$$w_t = (1 - \eta)b + \eta\tilde{z}_t \quad (2.12)$$

$$(1 - I_t)d_{t+1} = 0 \quad (2.13)$$

and

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0 \quad (2.14)$$

2.3.6 Optimal Government Policy

We focus on Markov perfect equilibria in which a benevolent government chooses the level of tax on debt to maximize households welfare subject to implementability conditions. The lump-sum tax is residually determined in a way to balance the per period government budget constraint.

If the country is in good financial standing in period t , $I_{t-1} = 1$, the value of continuing to service the external debt, denoted as $V^c(z_t, d_t)$, i.e., the value of setting $I_t = 1$, is given by

$$V^c(z_t, d_t) = \max_{C_t, d_{t+1}} \left\{ \frac{C_t^{1-\sigma} - 1}{1 - \sigma} + \beta \mathbb{E}_t V^g(z_{t+1}, d_{t+1}) \right\}$$

subject to the government budget constraint (5), the resource constraint (7), and two implementability constraints (11) and (12), where $V^g(z_{t+1}, d_{t+1})$ denotes the value of being in good financial standing. The value of being in bad financial standing in period t , denoted as $V^d(z_{t+1})$, is given by

$$V^d(z_t) = \max_{N_{t+1}} \left\{ \frac{(\tilde{z}_t N_t - v_t k)^{1-\sigma} - 1}{1 - \sigma} + \beta \mathbb{E}_t \left[\lambda V^g(z_{t+1}, 0) + (1 - \lambda) V^d(z_{t+1}) \right] \right\}$$

subject to equation (5), (11) and (12).

In any period t in which the country is in good financial standing, it has the option to either continue to service the debt obligations or to default. It follows that the value of being in good standing in period t is given by

$$V^g(z_t, d_t) = \max \left\{ V^c(z_t, d_t), V^d(z_t) \right\}$$

so the default set is

$$B(d_t) = \left\{ z_t : V^c(z_t, d_t) < V^d(z_t) \right\}$$

and the bond price schedule is

$$q(z_t, d_{t+1}) = \frac{1 - Pr[z_{t+1} \in B(d_{t+1})|z_t]}{1 + r^*}$$

2.4 Quantitative Exercise

I solve the model numerically using discrete state space value function iterations, see Appendix 6.1 for details. The model is calibrated to the Argentine economy in which one period corresponds to a quarter. The intertemporal elasticity of substitution, σ , is set at 2 which is in line with much of the related literature. The world interest rate, r^* , is set to 1 percent per quarter. The probability of reentry, λ , is calibrated so that the average exclusion period is 7.5 years.

The curvature parameter in the matching function, ι , is set to be 3.46 as in Pei and Xie (2017). I calibrate the fixed per unit vacancy cost, k , at 0.04 so that the model can reproduce a realistic quarterly unemployment rate of 7.8 percent, which is not far from the average unemployment rate of 8.3 percent from December 2002 to March 2018 period ¹⁰. The magnitude of vacancy cost corresponds to 7.5 percent of quarterly wages, which seems empirically plausible in emerging economies like Argentina given relatively high barriers to entry for firms.

The workers bargaining power, η , is set at 0.64, which is also the wage elasticity to labor productivity. Notice that in the current setup, the separation rate of worker-job pair is equal to one. As a result, wage does not depend on labor market tightness. I pick the value of $b = 0.30$ and the resulting replacement ratio is 40.72 percent that is close to Shimer (2005).

I assume that aggregate productivity follows an AR(1) process for the logarithm of z with an autocorrelation of 91.59 percent per quarter and a standard deviation of 2.71 percent. These moments imply that output is of 7.27 percent standard deviation. Following Arellano (2008), I assume that when the country is in bad standing, it loses any productivity above a certain threshold \bar{z} , i.e.,

¹⁰Data source: CEIC Data ().

$$z_t - L(z_t) = \begin{cases} z_t, & \text{if } z_t < \bar{z} \\ \bar{z}_t, & \text{if } z_t > \bar{z} \end{cases}$$

2.5 Discussion

Then I estimate \bar{z} to be 0.9. Finally, the subjective discount factor, β , is calibrated at 0.854. Together with the rest of the parameter values, the model produces the following three equilibrium implications: (i) the average debt-to-GDP ratio in periods of good financial standing is about 54.7 percent per quarter; (ii) the frequency of default is 2.7 times per century, and (iii) the average output loss is 11 percent per year conditional on being in financial autarky. Table 2.1-2.3 summarize the values of parameters and some other selected empirical and theoretical first and second moments ¹¹.

Table 2.1 Externally Calibrated Parameters

Parameter	Value	Description
r^*	0.01	risk-free return
λ	0.0333	prob. of reentry
σ	2	risk aversion
ρ	0.9159	autocorrelation
σ_ρ	0.0271	standard deviation of μ
k	0.04	vacancy cost

Table 2.2 Internally Calibrated Parameters

Parameter	Value	Description
β	0.854	discount factor
\bar{z}	0.90	post-default productivity
b	0.30	unemployment benefit
ι	3.46	matching parameter
η	0.64	worker's bargaining power

¹¹Note: All data are from Uribe and Schmitt-Grohe (2017), except the standard deviation of country remium is from Arellano (2008).

Table 2.3 Results and Comparison: Debt Market

Sources of statistics	Default frequency	Debt-to-Y ratio	Average spread	SD of spread	Corr of spread and Y	Corr of spread and TB-to-Y
Data	2.6	58.0	7.4	5.5	-0.64	0.72
Model	2.7	54.6	3.6	4.5	-0.47	0.74

From Table 2.3 we can see that the model only explains half of the observed average country premium in Argentina (3.5 percent versus 7.4 percent per year). As highlighted in Na et al. (2018), international lenders are risk-neutral so that the average country premium should approximately be the same as the average frequency of default. In this sense, there is no way for the model to explain both moments at the same time unless they are equal to each other in the data.

Another couple of unrealistic moments replicated by the model are the volatility and counter-cyclicality of the country premium. In my model, the default penalty is set similarly as in Arellano (2008) and Balke and Ravn (2016) in which the country loses any productivity above a certain threshold \bar{z} . Compared to the quadratic loss function specified by Chattterjee and Eyigungor (2012), it induces lower volatility of the country premium and reduces its correlation with output. Despite the shortfall, the positive correlation between the trade balance and the country premium does match closely with the data. The intuition, as highlighted by Uribe and Schmitt-Grohe (2017), is because international lenders would demand that the country at risk of default make an effort to improve its financial situation by at least paying part of the interest due.

Table 2.4 and 2.5 display the first and second moments of several labor market variables generated from the model. First note that the average job-finding rate is more than twice as large as the job-filling rate. Recall that the total measure of workers looking for jobs at the beginning of each period is one. The lack of accumulation of employment stock thus calls for a constantly tighter labor market to deliver realistic average unemployment level. This leads to an empirically implausible vacancy rate that is larger than one. In Balke and Ravn (2016), they circumvent this issue by introducing individual search cost. Also, my model fails to generate observed cyclical movements in terms of unemployment and vacancy. As Shimer (2005) suggests, this is because wage is too flexible

with respect to productivity. Intuitively, in good times when productivity is high, the wage level is also high so that firms have no incentive to create more vacancies. To solve the Shimer's puzzle, one way is to use the calibration strategy proposed by Hagedorn and Manovskii (2008). In particular, they argue that the flow value of unemployment activities should equal that of employment. This makes wage *de facto* rigid. Firms cannot adjust wage downward and thus reduce vacancy creation during the downturn driven by productivity shocks. However, the experiment with HMs calibration of the current model still couldn't generate sufficiently volatile job market flows. In our baseline model, unemployed workers are defined as those who fail to find a job in the labor market, which is largely distinct from the usual definition in the DMP model of people who are actively looking for jobs. The extension to a dynamic labor market is discussed in Section 4.

Table 2.4 Results: labor market

Sources of statistics	Average job-finding rate (monthly)	Average job-filling rate (monthly)	Average Replacement Ratio (percentage)
Model	0.74	0.31	40.71

Table 2.5 Summary Statistics: cyclicality from the calibrated Markov economy

Statistics	Productivity	Unemployment	Vacancy
	z	u	v
Standard deviation	0.065	0.007	0.043
Correlation Matrix	z	1	0.806
	u	-	-0.999
	v	-	1

Figure 2.1 reports the dynamics of the model economy around a typical default episode under optimal policy. This is obtained by stochastic simulations of the model for one million periods. The first 0.1 million periods are discarded and I compute the paths of the key variables of both debt and labor markets in three year windows around the time of default. Notice that, as predicted by the model, the lump-sum transfer, T_t , becomes negative at default. This is because when the government chooses to default, it immediately confiscates any payments of households to international lenders and returns the proceeds to households via income subsidies (Uribe and Schmitt-Groh,

2017). After default, as long as the country is in bad financial standing, the government has to impose a lump-sum tax so as to meet the need of financing the unemployment benefits.

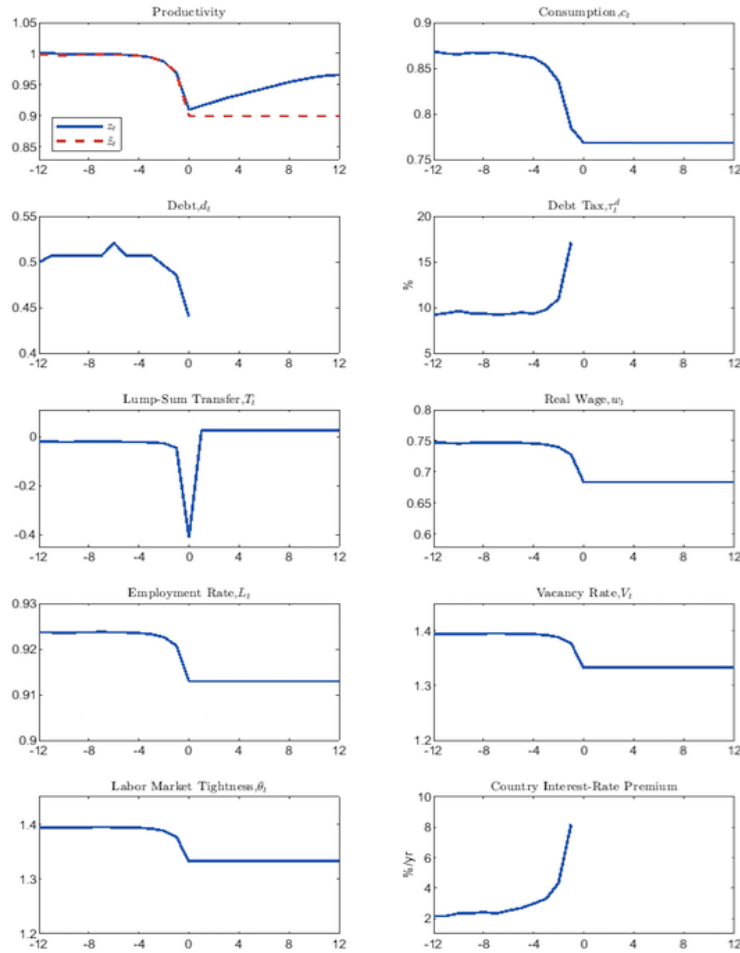


Figure 2.1 A Typical Default Episode Under Optimal Policy

In regard to the fiscal policy, the macroprudential type capital control tax increases the effective country interest-rate premium and helps private households internalize the pecuniary externality. The net stock of external debt is going down as the country is on the verge of the debt crisis though. Upon default, however, the unemployment increases less than half percent.

2.6 Extension

The baseline model assumes a static labor market which leads to two unsatisfactory outcomes. One is that the unemployment is not persistent. As a result, the model cannot generate plausible cyclical dynamics in terms of labor market flows. It also gives rise to an empirically unrealistic tight labor market. The other drawback is that the labor and debt market are disentangled in the sense that the dynamics of the former is purely driven by productivity shocks. To examine the interactions between the two markets, we need to extend our analysis to a dynamic labor market. As in Petrosky-Nadeau et al. (2018), the representative firm is now facing a different employment constraint. The firm's problem becomes

$$\max_{v_t} \mathbb{E}_t \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} (\tilde{z}_t N_t - w_t N_t - v_t k)$$

subject to

$$N_{t+1} = (1 - s)N_t + M(u_t, v_t)$$

and

$$v_t \geq 0$$

where $\Lambda_t = C_t^{-\sigma}$ and $\beta \frac{\Lambda_{t+1}}{\Lambda_t}$ denotes the stochastic discount factor of the firm inherited from the representative household. Firms intertemporal job creation condition is thus given by

$$\frac{k}{f(\theta_t)} - \mu_t = \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left\{ \tilde{z}_{t+1} - w_{t+1} + (1 - s) \left[\frac{k}{f(\theta_{t+1})} - \mu_{t+1} \right] \right\} \quad (2.15)$$

where μ_t is the Lagrangian multiplier associated with the constraint $v_t f(\theta_t) \geq 0$. PZK suggests that because of the occasionally binding vacancy constraint, there exists highly nonlinear dynamics that calls for a globally nonlinear algorithm to solve the model accurately, though they find in simulations it is very rare for the vacancy constraint to bind.

Also note that both sides of equation (15) depend on the tuple $\{z_t, d_t, N_t\}$, in which employment level N_t is a state variable. This not only makes it difficult for the numerical algorithm to converge, but could also induce complicated government non-commitment problem if we consider the unemployment benefit, b , as a policy instrument. Due to Nash bargaining, wage depends on the workers outside option. The strategic interaction between the government and the private sector in general would lead to multiple equilibria. In Pei and Xie (2017), they specify an exogenous wage setting to facilitate numerical convergence and generate enough unemployment variance. Although in this way, they admit that the macro channel emphasized in Hagedorn et al. () is muted since benefit policies does not affect wages.

2.7 Conclusion

We introduce one additional element into the Eaton and Gersovitz (1981) model: labor market is frictional. This elaboration on the standard quantitative sovereign debt model can equivalently be described as a DMP labor search and matching model with aggregate productivity shocks and with one extra ingredient: sovereign borrowing and default risk. We find that with static labor search and matching setup, the model is feasible to solve despite a number of new elements relative to earlier models with productivity shocks and virtually no propagation.

Our positive findings are that the incorporation of a frictional labor market can help the model generate plausible debt level at good times and overall default frequency that fits the data. It also enables the model to replicate a couple of first and second moments that are not targeted. Specifically, it explains the observed volatility of the country premium and its positive correlation with the trade balance. The simulation of a typical default episode under optimal fiscal policy shows that the imposed debt tax helps private households internalize the increased sensitivity of the sovereign spreads with respect to debt in the run up to default.

Our model can serve as a framework for analyzing various stabilization and social-insurance policies in the context of financial crisis. The fixed level of unemployment benefit during the recession or the recovery of the economy helps the household to smooth consumption, but comes at

two different kinds of cost: it deters firms vacancy creation by leveraging the workers outside option, and at the same time, increases the governments financing burden as it needs to maintain a balanced budget constraint even during the downturns. It would be interesting to extend our analysis to a dynamic frictional labor market with persistent unemployment and intertemporal job creation decisions by firms. In particular, Petrosky-Nadeau et al. (2018) recently explore the dynamics of endogenous rare disasters generated by the standard search model by solving it accurately with a globally nonlinear algorithm. The presence of an occasionally binding vacancy constraint would help propagate and amplify the effect of productivity shocks and default penalty on the fluctuations of unemployment and vacancies.

One dimension in which the baseline model is not empirically satisfactory is in its inability to simultaneously generate realistic average employment at good times and enough unemployment volatility over the business cycle frequency, even after one adopts the alternative calibration strategy proposed by Hagedorn and Manovskii (2008). Another drawback is the relatively low subjective discount factor. Future versions of the present settings ought to introduce long-term bond and possibly allow the government to optimally choose both the duration and level of unemployment benefit in a time-consistent fashion.

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2.9 Appendix

2.9.1 Algorithm and method

I use discrete state space value function iterations and Brute-force grid search method ¹² in MATLAB to numerically solve for V^c , V^d and V^g over $\mathbb{Z} \times \mathbb{D}$. Here are the algorithm in steps:

1. Use 200 grid points each to cover the values of z_t and d_t , respectively, so the state space is $\mathbb{Z} \times \mathbb{D}$, let π denote the transition probability matrix of z_t , the values of debt lie between 0 and 0.6 with equally spaced grid points. The transition probability matrix is estimated using the method proposed by Schmitt-Grohe and Uribe (2014). Alternatively, one can follow either Tauchen (1986) or Rouwenhorst (1995) to estimate it.
2. Specify values of parameters, construct two block arrays for d_0 and z_0 over \mathbb{Z} for each grid point in \mathbb{D} , respectively (the dimension is $200 \times 200 \times 200$), also construct a matrix for z over \mathbb{D} (the dimension is 200×200).
3. Initialize guesses (zeros) for value functions V^c , V^d and V^g on the grid over $Z \times D$, let $i = 0$.
4. Define indicator function $df = 1_{\{vc < vb\}}$, start with guesses of bond price

$$q = \frac{1 - \pi \cdot df}{1 + r^*}$$

5. For each d and z , compute V_{i+1} by brute-force grid search.
6. check whether ϵ is less than $TOL = 10^{-8}$.
7. Run simulations of the model under optimal policy for 1.1 million quarters and discard the first 0.1 million quarters.
8. Identify default episode that are of interest.

¹²The programming code has largely benefited from the textbook supplemental material from Uribe and Schmitt-Grohe (2017) and Na et al. (2018).

2.9.2 Calibration using Hagedorn and Manovskii (2008, HM)

Table 2.6 Updated Calibration Values of Parameters

Parameter	Value	Description
β	0.863	discount factor
\bar{z}	0.90	post-default productivity
b	0.35	unemployment benefit
ι	0.307	matching parameter
η	0.64	worker's bargaining power

Table 2.7 Results and Comparison: Debt Market

Sources of statistics	Default frequency	Debt-to-Y ratio	Average spread	SD of spread	Corr of spread and Y	Corr of spread and TB-to-Y
Data	2.6	58.0	7.4	5.5	-0.64	0.72
Model	2.6	56.2	3.6	6.3	-0.43	0.68

Table 2.8 Results: labor market

Sources of statistics	Average job-finding rate (monthly percentage)	Average job-filling rate (monthly percentage)	Average Replacement Ratio (percentage)
Model	9.19	4.40	91.36

In this section I report the calibration results from the baseline model using the strategy proposed by Hagedorn and Manovskii (2008). For values of parameters, four of them are updated: the subjective discount factor, the workers bargaining power, the matching parameter, and the level of unemployment benefit. The standard deviation of output of all times now becomes 1.43 percent, with the quarterly hiring cost to wage ratio of 7.22 percent and average labor market tightness (or vacancy rate) of 0.69. Table 2.6-2.9 summarize the rest results.

From Table 2.6, we can see that the HM calibration strategy gives rise to sizable improvement in terms of the variation of vacancy rate. The intuition, is that a commensurate value of unemployment benefit to wage rate makes workers indifferent between working on the market and staying at home.

Table 2.9 Summary Statistics: cyclicalities from the calibrated Markov economy

Statistics		Productivity	Unemployment	Vacancy
		z	u	v
Standard deviation		0.065	0.008	0.125
Correlation Matrix	z	1	-0.812	0.823
	u	-	1	-0.999
	v	-	-	1

As a result, small variations in productivity would cause large fluctuations in labor market flows.

However, the average unemployment rate now jumps up to an unrealistic level of 91.39%.

CHAPTER 3. A TWO-COUNTRY MODEL OF BANKING CRISIS

Modified from a manuscript to be submitted to the Journal of International Economics

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3.1 Abstract

Financially integrated economies observe a cross-country credit boom prior to financial recessions and a bust afterwards. This paper presents a two-country real business cycle model with banking sector where privately known intermediation efficiency of banks make them heterogeneous and gives rise to an interbank market. Overaccumulation of assets or low productivity in one country may lead to credit freeze in both financially integrated countries due to the existence of moral hazard and asymmetric information in the interbank market. A "sail together" financial integration may go into a "sink together" interbank credit freeze.

3.2 Introduction

Recent macroeconomic literature has emphasized on the propagation and amplification of random adverse financial shocks which primarily causes the banking crises, see, for example, Gertler and Kiyotaki (2010) and Gertler and Kiyotaki (2015). However, recent studies on banking crises has highlighted a close linkage between such crises and credit conditions as in Gorton (2012). Recent empirical research also corroborate the existence of typical patterns across diverse episodes. There are closed economy models of banking crises such as Gertler and Kiyotaki (2015) and Boissay et al. (2016). In a financially integrated world, banking crises may propagate and affect other countries. Banking crises are rare events, they follow credit-intensive booms, and involve with multiple nations occasionally. These stylized facts therefore call for an alternative approach of banking crises.

When countries are financially integrated, the interbank market equilibrium may be different than a closed economy. Moreover, the interbank crises may result from the states of other countries and therefore the threshold level of the states may also vary for crises. A financially integrated world may lead to spillover effects of banking crises from one country to another. These ideas lead to the research questions: how does the interbank market operate in a financially integrated two-country model? When do countries face inter-banking crises in an integrated world? Is there any spillover effect of interbanking crises from one country to another? In this paper, we seek for the answers of these questions under a two-country model.

The main finding of our paper is that when two countries are integrated through interbank lending, banking crises result from the procyclicality of bank balance sheets of either or both of the countries and may have spillover effects of interbank market freeze on each other. Countries get financially integrated to "sail together" and survive during recessions. However, due to over-accumulation by one country both countries may fall into an interbank freeze to "sink together". During expansions, credit supply increases pushing both the corporate and interbank lending rates down. With lower rates the agency problem in the interbank market gets aggravated leading to contractions in interbank funding. Financial integration may lead to a disastrous interbank market freeze if total credit boom is larger than the productive use of loans.

In our model, we follow and extend the real business cycle setup with heterogeneous banks as in Boissay et al. (2016). However, we have a two-country world where the countries are symmetric. Banks are heterogeneous due to the privately observed intermediation efficiency. They can obtain funds from the depositor or the interbank market and lend to homogeneous good producing firms. Due to the usual agency problem in the interbank market, banks can borrow and divert funds to lower return assets and such diversion cannot either be traced or recovered by the lending banks. Less efficient banks have more incentive to divert and diversion depends on the rate of returns in the economy. To restrict such diversion interbank market has a borrowing capacity resulting from the incentive compatibility constraint. However, lower return in the corporate loans market increases the incentive for diversion and therefore market responds through lowering the

borrowing capacity and eventually the market goes into a freeze. In a financially integrated world, freeze in one country leads to freeze in another country for lower rates of returns.

The mechanism that leads to a banking crises in a two-country model is as follows. A sequence of favorable, productivity shocks lead to an expansion of credit. The more efficient banks borrow from the less efficient ones to expand their corporate loan operations. The size of the overall banking sector increases and the economy booms. However, when the aggregate productivity growth goes down to converge to its long-run average course, the demand for corporate loans and the corresponding rate goes down. As a result, the interbank lending rate goes down resulting into more incentive for less efficient banks to borrow and divert funds. Therefore, counterparty risk goes up since the identity of these diverting banks are unknown, which in turn leads to the decline in interbank lending activities and the market goes into a freeze. When two countries are integrated through interbank lending, a market freeze in one country leads to a freeze in another country as well. Either two countries have an operating interbank market or both goes into a freeze. Excessive credit creation by one country triggers a two-country wide credit freeze. The countries may also go into a freeze from over low productivity in either or both of the countries.

We calibrate the interbank market equilibrium of the model and assess its quantitative properties. Potentially, the model can generate banking crises in one country due to an endogenous factor-overaccumulation of assets or an exogenous factor- low productivity or a combination of both the endogenous and exogenous factors in a period. Moreover, the banking crises in one country shows a spillover effect to another country, i.e., a country that could have a well-operated interbank market without any integration, falls into interbank market freeze due to either of the factors of another country after any financial integration. Thus, as in data, banking crises in one country are closely linked to asset accumulation and productivity of not only its own but also another country to which the country is financially integrated. However, we find that this story largely depends on how the countries are integrated. In particular, the narrative is applicable if households are allowed to deposit in domestic country only and countries are integrated through only the interbank market and firms can only borrow capital from the domestic banks.

The paper proceeds as follows. Section 3.2.1 depicts some related literature of macroeconomic models with financial frictions, international spillover effect, and contagion. Section 3.3 documents the facts about financial recessions of 140 OECD countries for the period of 1870-2008 as in Boissay et al. (2016). Section 3.4 describes the basic model with representative household, representative firm, and heterogeneous banks. Section 3.5 characterizes the interbank market equilibrium for both closed and financially integrated two countries which are integrated in different dimensions. In Section 3.6, we explain the calibration parameters and the results of the interbank market equilibrium to alternative state spaces. Section 3.7 presents the concluding remarks.

3.2.1 Related Literature

This paper is related to the macroeconomic literature with financial frictions. For instance, Bernanke et al. (1999), Gertler and Karadi (2011), and Jermann and Quadrini (2012) show how financial market frictions can amplify the financial shocks and generate long-lasting recessions. However, the models are linearized in this line of work and this paper departs from this approach to characterize the important and critical nonlinearities in the mechanism that drives the financial crises. The important difference is to show that the boom-bust cycle as an outcome of credit-driven endogenous factors given only one standard deviation to the productivity level.

In this respect, this paper is related to the literature featuring powerful amplification mechanism¹ resulting from financial frictions, e.g., Brunnermeier and Sannikov (2012) and He and Krishnamurthy (2012). This paper is also related to the literature on sudden stops in emerging market economies. For example, as in Mendoza and Smith (2005) and Mendoza (2010), we find that a plausible magnitude of productivity shock can trigger a crisis if the agent is highly leveraged. These are also open economy models similar to our model. However, our paper departs in that it does not assume an exogenous interest rate, instead it is the endogenously generated market equilibrium interest rate that plays a central role in the interbanking activities.

¹Financial crises are inherently nonlinear events, often featuring sudden plummet in asset prices and sharp collapse in output, while with a slow recovery rate. The most recent approach to capture this nonlinearity is to allow for an occasionally binding constraint.

Our paper is also related to the financially contagion literature for international business cycle models. For example, Calvo (1995) and Chang and Velasco (1998), among others, study the interaction of the banking system and currency markets in a crisis. Also Allen and Gale (2000) study the financial contagion as an outcome of liquidity preference shock.

In a closely-related paper, Perri and Quadrini (2018) also shows that in a two-country model with financial frictions, a global liquidity shortage induced by pessimistic self-fulfilling expectations can quantitatively generate longer and deeper recessions following credit booms. The main differences between theirs and our work are that they focus on balance sheet effect from the private sector, i.e., the role of balance sheets in constraining borrowers from spending when financial markets are imperfect, while our work highlights the role of the wholesale banking sector at the heart of the recent financial crisis in which credit dried up and market froze².

A recent work³ of Gertler et al. (2019) incorporates banking panics into a standard New Keynesian model and captures both qualitatively and quantitatively the effects of financial collapse on investment, output, and employment. They show that a credit boom can increase financial instability of the system, as in Boissay et al. (2016). The sudden and discrete nature of the financial collapse characterized by the paper is induced without observing large exogenous productivity shocks, either. Instead, banks that hold imperfectly liquid long term assets and issue short term debts facing liquidity mismatch are vulnerable to panic failure of household to roll over deposits. A plausible magnitude of negative capital quality shock reduces the value of banker's capital, which either brings banks immediately into insolvency, or leads to a self-fulfilling rollover crisis in which the liquidation of assets at firesale prices forces banks into default, similar to the Cole and Kehoe (2000) model of sovereign default but contrast with the "early withdrawal" mechanism in the Diamond and Dybvig (1983) model⁴. One of their paper's contributions to the macro-finance literature is that they show that banking panics (or bank runs) are quantitatively more important than non-

²See Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Curdia and Woodford (2010) for papers that incorporate banking and Iacoviello (2005), Eggertsson and Krugman (2012), and Guerrieri and Lorenzoni (2017) for papers that included household debt.

³See also, Gertler and Kiyotaki (2015) and Gertler et al. (2016).

⁴The traditional bank run models, starting from Diamond and Dybvig (1983), focus on the retail banking sector in which a "sequential service constraint" takes in effect to generate runs, i.e., only households who get to banks early will have their money back, a *first-come, first-serve* scenario. However, recent bank run mostly occurs in the

linearities coming from occasionally binding constraints, as in Brunnermeier and Sannikov (2012) and He and Krishnamurthy (2012). They also managed to endogenize the probability of bank runs by relating it to macroeconomic fundamentals. However, there are three main weaknesses associated with Gertler et al. (2019). First, it is not clear what the negative capital quality shocks stand for in the model if we are trying to understand the interactions between the financial market and the real economy. Second, the paper tends to replicate what happened during the 2007-08 financial crisis but fails to match the exact timing of when the crisis occurred. This issue may be addressed by incorporating some adjustment cost. Third, their model generates a counterfactual faster recovery rate from the recession than in the data. The lack of persistency may be resolved by assigning an equally important role to the real side of the economy in which household deleverages as well in the crisis.

3.3 Financial Recession Facts

Based on data from Jorda et al. (2011), Jorda et al. (2013) and Schularick and Taylor (2012) of 140 OECD countries from 1870 to 2008, there are two facts regarding the financial recessions (Fact 1 and Fact 2). Fact 3 is built on the data from Bank for International Settlements (BIS).

- Fact 1: Financial recessions are rare events

Financial recessions are rare events in comparison to other recessions. From data collected of 196 recessions of 140 OECD countries, the average probability of a financial recession is 2.36 percent whereas other recessions have the probability of 8.93 percent on average.

- Fact 2: Financial recessions follow credit booms

Financial recessions do not hit at a random. Instead they break out following credit booms. Credit is 3.25 percent above the trend in the peak year prior to a financial recession in comparison to only 0.61 percent above the trend in case of other recessions.

wholesale banking (or interbank market) sector, such as the failure of Continental Illinois in 1984 and the collapse of Lehman Brothers in 2008.

- Fact 3 International financial recessions follow international credit booms

International financial recessions follow an increase of international credit by more than 8 percent from the average in the peak year prior to the recession and during the financial recession the international credit decrease by almost 13 percent from the average.

3.4 The Baseline Model

This section explains a symmetric two-country ($j = 1, 2$) real business cycle model with a banking sector. Each country is populated with one risk averse representative household, one risk-neutral representative competitive firm, and risk-neutral, heterogeneous and competitive banks with a mass of one. For ease of notation, we exclude the country subscripts for model characterizations. They will be re-introduced when necessary.

3.4.1 The Representative Household

There is an infinitely lived, risk averse, representative household in each of the countries. Household in each country has preferences over consumption, c , with an utility function

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} u(c_{t+\tau}) \quad (3.1)$$

where $u(\cdot)$ satisfies the regularity conditions, i.e., $u'(c) > 0, u(c) < 0, u'(0) = \infty, u'(\infty) = 0$. The subjective discount factor of the household is denoted as $\beta \in (0, 1)$ which is common in both countries, and $\mathbb{E}_t(\cdot)$ denotes the expectation operator. The households starts each period t with an individual asset a . Aggregate assets are denoted A . We can think of a as either bank equity or bank deposit and the composition of a is indeterminate due to the lack of friction between the household and the banks. Suppose now that a is the bank deposit and the corresponding gross return on deposits is r . Along the convention of macro-finance literature as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), we assume that household cannot finance the firms directly due to the frictions between them. I also assume that the supply of labor is inelastically one unit by the

household who earns the market wage rate w for labor, obtains firm profit π , and receives a lump-sum transfer χ corresponding to the financial intermediation cost of the banks (more explanation will be given later in the banking sector section). A representative household therefore maximizes her utility (3.1) by choosing the consumption and level of assets subject to the following budget constraint

$$c + a' = ra + w + \pi + \chi \quad (3.2)$$

3.4.2 The Representative Firm

This is a two-country model where the firm lives only for a period and produces a homogeneous good that can be either consumed or invested. To produce the good, the firm needs to hire capital k and labor h . The production function $zF(k, h)$ follows a constant returns to scale technology and satisfies the standard Inada conditions. The countries may differ due to their country-specific total factor productivity (TFP) shocks z . However, TFP in both countries are assumed to follow an AR(1) process:

$$\log z' = \rho_z \log z + \epsilon' \quad (3.3)$$

where the persistence parameter $|\rho_z| < 1$ and the innovation ϵ is normally distributed with mean zero and standard deviation σ_z . At the beginning of period t , ϵ is realized, before the firm decides on the demand of capital k and labor h . At the beginning of the period t , firm is born without any resources and depends on banks to borrow capital k at a gross corporate loan rate R . Capital k depreciates at rate $\delta \in (0, 1)$. At the end of the period, the firm repays the corporate loan. For production, the firm also rents labor from the household and in return pays w per unit of labor. The firm chooses capital k and labor h for production to maximize its profit

$$\pi = zF(k, h) + (1 - \delta)k - Rk - wh \quad (3.4)$$

3.4.3 The Banking Sector

The banking sector plays a critical role in our model due to two salient features. First, there exists heterogeneity in the intermediation skill among banks, i.e., some banks have more efficient intermediation skills than others. This heterogeneity among banks creates an interbank market where the least efficient banks lend to the most efficient ones. Second, following the corporate finance literature as in Tirole (2006), there is a moral hazard problem in the banking sector along with asymmetric information, which essentially impede the interbank market functioning. Overall, banks perform both the retail banking and wholesale banking. On the retail side, they collect deposits from the household and lend to firms, while on the wholesale side, they issue interbank claims to reallocate assets among the interbank market in which low-skilled banks, during normal times, lend to highly-skilled ones.

3.4.3.1 Banks

There is a continuum of risk-neutral, competitive banks in both countries. They are born at the end of period $t - 1$ and only live for one period, i.e., banks dissolve at the end of period t . Each bank, after birth, collects deposits a from the household. They then become heterogeneous by drawing a random, bank-specific, intermediation skill p . Henceforth, we denote bank p as bank with intermediation skill p . The skill p is distributed over the interval $[0,1]$ with cumulative distribution $\mu(p)$, satisfying $\mu(0) = 0$, $\mu(1) = 1$, and $\mu'(p) > 0$. If the bank p decides to lend to the firm, then at the end of the period there is an intermediation cost $(1 - p)R$ per unit of corporate loan, so that the bank p earns an effective gross return of pR from corporate loan.

We abstract away from any deadweight loss concerns and assume that the household receives a lump-sum rebate χ from the bank that is equal to the amount of the intermediation cost. These intermediation costs may arise from the proposition, screening, and monitoring while originating or servicing the loan. The banks can also invest into an outside project with a constant gross return γ . To fix the idea, we refer this outside option as the storage technology which is at least as good as just letting the good depreciate, i.e., $\gamma \geq 1 - \delta$.

Due to heterogeneity of banks in terms of intermediation skill p , in each country there is an intra-periodic interbank market, in which low p banks lend to the high p ones at gross rate ρ . In equilibrium, this interbank loan rate must be lower than the corporate loan rate R , otherwise all banks would find it optimal to lend in the interbank market at a higher rate ρ than lending to firms at lower corporate loan rate R . Similarly, the storage return γ must be lower than the interbank lending rate ρ , otherwise all banks would find it optimal to store and not participate in the interbank lending market. Banks p take the interbank lending rate ρ and the corporate lending rate R as given and decides whether, and how much to borrow or lend.

Let Φ be the borrowing per deposit by a borrowing bank p , where Φ also denotes the publicly observable and endogenous market funding ratio. If a bank p decides to borrow Φ (per unit of deposit) from other banks at cost ρ then gets a gross unit return on deposits equal to $pR(1+\Phi) - \rho\Phi$, and lends $1 + \Phi$ (per unit of deposit) to the firm for gross return pR . If a bank decides to lend to other banks, instead, then the gross return on deposits is just ρ .

Gross return on deposits for bank p is then

$$r(p) \equiv \max\{pR(1 + \Phi) - \rho\Phi, \rho\} \quad (3.5)$$

Bank p chooses to be a borrower in the interbank market when

$$pR(1 + \Phi) - \rho\Phi \geq \rho \Leftrightarrow p \geq \bar{p} \equiv \frac{\rho}{R} \quad (3.6)$$

Inequality (3.6) imposes a participation constraint on bank p to be a borrower, not a lender in the interbank market. Banks with $p < \bar{p}$ are less efficient in intermediation and hence lend to the more efficient banks $p \geq \bar{p}$, and the marginal bank \bar{p} is indifferent between the two options. In a frictionless world, $\bar{p} = 1$ since all banks with $p < 1$ would lend to the most efficient one with $p = 1$ in that case. The economy would have reached the first-best allocations then. There are two frictions prevalent on the interbank market which prevent the economy from reaching their first-best allocation: moral hazard and asymmetric information.

3.4.3.2 Moral Hazard

We assume that the creditors cannot trace the proceeds of the storage technology and therefore cannot seize those diverting funds. This in turn leads to the fact that the borrowing banks may choose to renege on their interbank debt contracts and walk away from the lenders. Therefore, the interbank loan contracts are not enforceable. If bank p chooses to walk away with $(1 + \Phi)a$ and invests in the storage technology, gets $\gamma(1 + \theta\Phi)a$ as payoff, where $\theta \in [0, 1]$ is the cost for walking away from the interbank market after borrowing (the higher θ , the larger the cost). Such opportunistic behavior is referred as "diversion", see, Hart (1995) and Burkart and Ellingsen (2004).

Corporate finance literature (e.g., Tirole 2006) refers to the diversion as a standard moral hazard problem: (i) the higher Φ also increases the gain from diversion, and the diversion opportunity cost increases with (ii) bank efficiency p and (iii) the spread between the corporate lending rate R and the return on the storage technology γ . The last feature, which implies that as the corporate lending rate goes down, banks have an incentive to walk away from the interbank market, is consistent with the recent empirical evidence of taking high risks by the banks amid low interest rates, see, for example, Maddaloni and Peydro (2011) and Jimenez et al. (2014).

3.4.3.3 Asymmetric Information

The intermediation skill of the banks are private information. Ex-ante lenders cannot observe them and also cannot verify them ex post. Therefore, lenders ignore any private incentives of the borrowers to divert funds. This makes the interbank loan contracts the same for all banks, i.e., neither does the market funding ratio Φ nor the interbank lending rate ρ depends on p . However, the lenders want to deter the borrowers from diverting. The way to deter the borrowers from diverting is to limit the quantity of funds to the borrowers so that the least efficient banks with $p < \bar{p}$ have no incentive to pretend as high-efficiency bank and divert:

$$\gamma(1 + \theta\Phi) \leq \rho \quad (3.7)$$

This is the incentive compatibility constraint, which, eventually, sets a limit for Φ , i.e., the market funding ratio above which no bank would like to lend. Hence, it can also be regarded as the lenders' funding tolerance. At the optimum, this incentive compatibility constraint binds and the borrowing banks utilize the full borrowing capacity:

$$\Phi \equiv \frac{\rho - \gamma}{\gamma\theta}$$

The borrowing capacity Φ increases with the interbank lending rate ρ . With a higher ρ , banks lend more, and only more efficient banks with high p keep demanding a loan. Lenders have an incentive to tolerate a higher market funding ratio Φ since more efficient banks have lower incentive to divert. Therefore, there is a positive selection effect on the borrowers caused by an increase in the interbank rate. Symmetrically, there is a detrimental effect on incentives caused by a decrease in the interbank rate. In the limit, when the interbank rate is equal to the return on storage ($\rho = \gamma$), the demand is null ($\Phi = 0$) since no borrowers can commit themselves to repay.

3.5 Interbank Equilibrium

The interbank market equilibrium largely depends on whether the country is a closed economy or integrated with other countries. We will first characterize the equilibrium of the banking sector in one country without integration (see Boissay et al. 2016 for full details). Then, we introduce different dimensions of integration between the two countries and characterize the corresponding interbank market equilibrium.

3.5.1 Markets Without Integration

3.5.1.1 Interbank Market Clearing

In interbank market equilibrium of two countries without integration, the rate ρ clear the markets. We seek for an equilibrium where the return on storage is lower than the interbank rate ($\gamma < \rho$) so that the trade takes place ($\Phi > 0$). In a world of two countries without any integration,

a mass of $\mu(\bar{p})$ banks lend and the complement mass $1 - \mu(\bar{p})$ of banks borrow Φ per unit of deposit. The market clearing condition thus becomes

$$\underbrace{A\mu\left(\frac{\rho}{R}\right)}_{\text{Supply}} = \underbrace{A\left[1 - \mu\left(\frac{\rho}{R}\right)\right]}_{\text{Demand}} \underbrace{\frac{\rho - \gamma}{\theta\gamma}}_{\text{Intensive Margin}} \quad (3.8)$$

$$\Leftrightarrow R = \Psi(\rho) \equiv \frac{\rho}{\mu^{-1}\left(\frac{\rho - \gamma}{\rho - \gamma(1 - \theta)}\right)}$$

The interbank market clearing condition shows that as the interbank rate ρ increases so does the aggregate supply, while there are two opposite forces in the aggregate demand. On the one hand, an increase in the interbank rate causes lower aggregate demand because fewer borrowers demand funds at higher rate, which is the extensive margin effect. On the other hand, a higher interbank rate increases the borrowing capacity and thus the aggregate demand of funds, which is the intensive margin effect. At the aggregate level, when there are more borrowers, i.e., when ρ is small enough the latter effect more than offsets the earlier effect. It follows that when the interbank rate ρ is small, the aggregate demand curve bends backward and increases with ρ . The function $\Psi(\rho)$ is strictly convex and there exists a threshold $\bar{R} \equiv \Phi(\bar{\rho})$ for the corporate loan rate R above which there are two equilibria with trade and below which there is no equilibrium with trade. In the latter case, there exists a cutoff $\bar{p} = \gamma/R$ such that banks with $p < \bar{p}$ store, while banks with $p > \bar{p}$ borrow in the interbank market. We refer to such no-trade equilibrium as a banking crises. However, such an equilibrium is Pareto-dominated by the trade equilibrium and therefore we rule out this equilibrium by assuming that banks always coordinate in the trade equilibrium.

3.5.1.2 Aggregate Corporate Loan Market Clearing

The aggregate supply of corporate loans depends on the existence of the interbank market operations. On the one hand, when interbank trades take place, the supply of corporate loans is A , i.e., all bank assets are channeled to the firms. On the other hand, when the interbank market is frozen, banks with $\rho < \gamma/R$ use the storage technology and the aggregate supply of corporate

loan is $[1 - \mu(\gamma/R)]A$. The demand of aggregate corporate loan is determined through solving for capital, k from firm's profit maximization problem. Thus, the aggregate corporate loan clearing condition is

$$f_k^{-1}\left(\frac{R + \delta - 1}{z}\right) = \begin{cases} A, & \text{for equilibrium with trade} \\ A[1 - \mu(\frac{\gamma}{R})], & \text{otherwise} \end{cases} \quad (3.9)$$

where $f_k(k) \equiv \partial F(k, 1)/\partial k$.

3.5.1.3 Interbank Loan Market Freeze

The interbank loan market freezes if and only if

$$A > \bar{A} \equiv f_k^{-1}((\bar{R} + \delta - 1)/z) \Leftrightarrow z < \bar{z} \equiv (\bar{R} + \delta - 1)/f_k(A)$$

otherwise, the interbank loan market operates. \bar{A} is the so-called "absorption capacity" of the banks. This is the maximum quantity of assets that the banks can allocate efficiently. Above this threshold there is no trade in the interbank market because of the counterparty fears. As shown in Boissay et al. (2016), this threshold has an equivalent counterparty in terms of TFP shocks \bar{z} , below which the interbank market freezes. Market freezes thus result either from the overaccumulation of assets by the household or from an adverse productivity shock that reduces banks' absorption capacity, or from a combination of both endogenous and exogenous factors.

3.5.2 A Two-country World with Perfect Integration

In a closed economy model the monitoring costs $(1 - p)R$ are rebated back to the households to abstract from any wealth effects. In a two-country model, in which one country's savings may end up for deposit and loans in another country, we will assume that these transfers go to the source of savings. Suppose all capital markets and labor markets are integrated in a symmetric two-country world. The environment is then equivalent to a closed economy with two firms. Only

one country will produce by renting the entire world's capital and hiring the total labor supply of the two countries. This case is trivial.

3.5.3 A Two-country World without Labor Mobility

Now we assume that households can work only at the domestic firms. Under this restriction, firms in both countries will operate and produce. Under integration, deposits made by household and taken by banks, and loans made by banks may differ. Let A_j continue to be the aggregate deposits made by households of country j , let D_j denote the deposits taken by a (ex-ante identical) bank of country j , and k_j denote the loan (that equals to the capital stock) taken by the representative firm in country j .

We consider two alternative environments in which cross-border deposits are (1) permitted or (2) prohibited. In each of these, we in turn consider cases in which (i) cross-border interbank as well as corporate loans are permitted, and (ii) only corporate loans are permitted, and (iii) only interbank lending is permitted.

3.5.3.1 With Cross-border Deposit Mobility

We assume now that the depositors and the banks in either country are free to make and take cross-border deposits.

(i). With Open Interbank and Corporate Loan Markets

In this case, banks can lend to both banks and corporate firms of either country. From the perspective of depositors, all banks are then multinational and ex-ante identical. The total deposits of the two countries $A_1 + A_2$ will be equally deposited at all banks, now with a mass of two. Thus the deposits taken by each bank will be

$$D_j = D = \frac{A_1 + A_2}{2} \quad (3.10)$$

Under interbank trade the interest rate ρ will be common across the two countries. Furthermore, the firms' borrowing rate R will also be common. Then, with inelastic labor supply fixed at unity, the two countries' capital ratios is given by

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2 \quad (3.11)$$

The market clearing condition for the corporate loan market can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1} \left(\frac{R + \delta - 1}{z_j} \right) = \begin{cases} \sum_j A_j, & \text{for equilibrium with trade} \\ [1 - \mu(\frac{\rho}{R})] \sum_j A_j, & \text{otherwise} \end{cases} \quad (3.12)$$

where the first case holds if and only if $R > \bar{R} > \Psi(\bar{\rho})$.

When interbank trade exists, the total supply of interbank loans will equal the total demand

$$2D\mu\left(\frac{\rho}{R}\right) = 2D\left[1 - \mu\left(\frac{\rho}{R}\right)\right] \frac{\rho - \gamma}{\theta\gamma} \quad (3.13)$$

(ii). With Open Corporate Loan Markets Only

Let us continue to assume that that household can deposit in either country's banks. While banks cannot make cross-border interbank loans, the firms can still borrow from banks across the border. Banks can still lend to other domestic banks and therefore the interbank loan market equilibrium must satisfy

$$D\mu\left(\frac{\rho_j}{R}\right) = D\left[1 - \mu\left(\frac{\rho_j}{R}\right)\right] \frac{\rho_j - \gamma}{\theta\gamma}$$

where we have incorporated the fact that with open corporate loan market R is common across the two countries. The above in turn implies that ρ will also be common and so will be the per unit deposit rate. Once again, from the perspective of the depositors, banks in the two countries are fully multinational. As in the previous case with complete banking sector integration, the deposits are

$$D_j = D = \frac{A_1 + A_2}{2}$$

The two countries' capital ratios will follow as

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2$$

The market clearing condition for the corporate loan market can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1} \left(\frac{R + \delta - 1}{z_j} \right) = \begin{cases} \sum_j A_j, & \text{for equilibrium with trade} \\ [1 - \mu(\frac{\gamma}{R})] \sum_j A_j, & \text{otherwise} \end{cases} \quad (3.14)$$

where the first case holds if and only if $R > \bar{R} > \Psi(\bar{\rho})$.

(iii). With Open Interbank Market Only

We continue to assume that household can deposit in either country's banks. While banks can make interbank loans internationally, firms can only borrow from their country's banks. The banks are no-longer multinationals, instead, their opportunity sets now diverge. Hence, the deposits they attract are no longer going to be identical. They will be determined by the equilibrium as described below.

First note that two arbitrage conditions must hold. First, deposit rates offered by banks must be equal across the two countries. Second, under interbank trade, no storage will occur in either country.

Lemma 3.5.3.1. *In an interbank trade equilibrium the corporate loan rates in both countries are equalized.*

Proof. Let $\Psi_j \geq 0$ be the publicly known interbank borrowing (per unit of deposit) in country j . Irrespective of Ψ_j , the cutoff rule for the marginal lender/borrower bank is

$$\bar{p}_j = \frac{\rho}{R_j}$$

Since household can deposit in either country, all banks (in both countries) must offer the same deposit rate

$$r = R_j \int_{\frac{\rho}{R_j}} p \frac{d\mu(p)}{1 - \mu(p)}$$

Hence, $R_j = R$ for both country j . □

This in turn requires that

$$k_1 = \left(\frac{z_1}{z_2} \right)^{\frac{1}{1-\alpha}} k_2$$

with no storage occurs under interbank trade, that is

$$k_1 + k_2 = A_1 + A_2$$

Finally, since the incentive compatibility constraint holds with equality, i.e., $\gamma(1 + \theta\Psi) = \rho$, in both countries the borrowing capacity, $\Psi = \frac{\rho - \gamma}{\gamma\theta}$ must be the same. Within each country the mass of banks lending in the interbank market, $\mu\left(\frac{\rho}{R}\right)$, is the same. Also the borrowing per unit of deposits for the banks making corporate loans Psi is the same. This implies that

$$k_j = D_j$$

Banks in each country attract an amount of deposit that clears the domestic interbank lending as well as corporate loans. Hence, once the deposits are made, there is no need for cross-border interbank or corporate loans.

The banking sector equilibrium within each country follows the closed economy, which leads to the following proposition.

Proposition 3.5.3.1. *Interbank trade exists in both countries if and only if*

$$\sum_{j=1,2} f_{k_j}^{-1} \left(\frac{\bar{R} + \delta - 1}{z_j} \right) \geq \sum_{j=1,2} A_j$$

The converse of this result is that a banking sector freeze must occur simultaneously across the two countries. However, the deposit rates will still be the same across the two countries. This is only possible if

$$r = R_j \left[\frac{\gamma}{R_j} \mu \left(\frac{\gamma}{R_j} \right) + \int_{\frac{\gamma}{R_j}} p \frac{d\mu(p)}{1 - \mu(p)} \right]$$

for both country j . Once again $R_j = R$ for $j = 1, 2$ must hold and the capital used by firms must follow the arbitrage condition as in (3.11).

Corollary 3.5.3.1. *Under a banking sector freeze, the equilibrium capital allocations and corporate loan rate $\{k_1, k_2, R\}$ are given by*

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1} \left(\frac{R + \delta - 1}{z_j} \right) = \left[1 - \mu \left(\frac{\gamma}{R} \right) \right] \sum_j A_j$$

$$k_1 = \left(\frac{z_1}{z_2} \right)^{\frac{1}{1-\alpha}} k_2$$

In addition, in an interbank trade equilibrium, the equilibrium rate $\rho(R)$ continues to be determined by (3.13) with $2D$ on its both sides now trivially replaced by $\sum D_j$

$$\left(\sum_{j=1,2} D_j \right) \mu \left(\frac{\rho}{R} \right) = \left(\sum_{j=1,2} D_j \right) \left[1 - \mu \left(\frac{\rho}{R} \right) \right] \frac{\rho - \gamma}{\theta \gamma} \quad (3.15)$$

3.5.3.2 Without Cross-border Deposit Mobility

Banks in each country now take only domestic deposits

$$D_j = A_j \quad (3.16)$$

However, their asset side has no cross-border restrictions. We now sequentially consider the three alternatives.

- (i). With Open Interbank and Corporate Loan Markets

The banking sector equilibrium in this case is fairly similar to the previous first case, except for the size of the banks' balance sheets. Banks now have access to only domestic pool of deposits. The amount of loans therefore have to balance accordingly. With common corporate loan rates in the two countries:

$$k_1 = \left(\frac{z_1}{z_2}\right)^{\frac{1}{1-\alpha}} k_2$$

must hold. However, now we have $k_j \neq D_j = A_j$. That is, firm in country j may be borrowing from the other country and vice versa. When interbank trade exists, a common $\rho = \Psi^{-1}(R)$ emerges from (3.15)

$$\left(\sum_{j=1,2} A_j\right) \mu\left(\frac{\rho}{R}\right) = \left(\sum_{j=1,2} A_j\right) \left[1 - \mu\left(\frac{\rho}{R}\right)\right] \frac{\rho - \gamma}{\theta\gamma}$$

And the world-economy will jointly experience a credit crunch if and only if

$$\sum_{j=1,2} f_{k_j}^{-1}\left(\frac{\bar{R} + \delta - 1}{z_j}\right) < \sum_j A_j$$

where $\bar{R} = \Psi(\bar{\rho})$ The corporate loan market clearing can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1}\left(\frac{R + \delta - 1}{z_j}\right) = \begin{cases} \sum_j A_j, & \text{for equilibrium with trade} \\ [1 - \mu(\frac{\gamma}{R})] \sum_j A_j, & \text{otherwise} \end{cases}$$

where the first case holds if $R > \Psi(\bar{\rho})$.

(ii). With Open Corporate Loan Markets Only

Here, banks can make cross-border loans to firms and therefore R must be equal in the two countries. Banks can make within country interbank loans. Let ρ_j denote within country interbank rates. The interbank loan market equilibrium can now be expressed as

$$A_j \mu\left(\frac{\rho_j}{R}\right) = A_j \left[1 - \mu\left(\frac{\rho_j}{R}\right)\right] \frac{\rho_j - \gamma}{\theta\gamma}$$

Obviously, $\Psi : \rho \rightarrow R$ is common for both countries.

Proposition 3.5.3.2. *With open corporate loan markets, within country interbank lending rates are equal.*

With common corporate loan rates in the two countries

$$k_1 = \left(\frac{z_1}{z_2} \right)^{\frac{1}{1-\alpha}} k_2$$

must hold. However, as in the previous case, $k_j \neq A_j$. And the world economy will jointly experience a credit crunch if and only if

$$\sum_{j=1,2} f_{k_j}^{-1} \left(\frac{\bar{R} + \delta - 1}{z_j} \right) < \sum_j A_j$$

where $\bar{R} = \Psi(\bar{\rho})$. The corporate loan market clearing can be expressed as

$$\sum_j k_j = \sum_{j=1,2} f_{k_j}^{-1} \left(\frac{R + \delta - 1}{z_j} \right) = \begin{cases} \sum_j A_j, & \text{for equilibrium with trade} \\ [1 - \mu(\frac{\gamma}{R})] \sum_j A_j, & \text{otherwise} \end{cases}$$

where the first case holds if $R > \Psi(\bar{\rho})$. In the second case, the amount stored in each country equals $A_j \mu(\frac{\gamma}{R})$.

(iii). With Open Interbank Market Only

Under interbank trade, once again, ρ will be common in both countries. However, corporate loan rates may no longer be equal. The interbank loan market equilibrium can now be expressed as

$$\underbrace{A_1 \mu(\bar{p}_1)}_{\text{Domestic Supply}} + \underbrace{A_2 \mu(\bar{p}_2)}_{\text{Foreign Supply}} = \underbrace{A_1 [1 - \mu(\bar{p}_1)] \Phi_1}_{\text{Domestic Demand}} + \underbrace{A_2 [1 - \mu(\bar{p}_2)] \Phi_2}_{\text{Foreign Demand}}$$

Using $\Phi_1 = \Phi_2 = \Phi = \frac{\rho - \gamma}{\gamma \theta}$, and $\bar{p}_j = \frac{\rho}{R_j}$, and market clearing condition becomes

$$A_1\mu\left(\frac{\rho}{R_1}\right) + A_2\mu\left(\frac{\rho}{R_2}\right) = \left[A_1\left(1 - \mu\left(\frac{\rho}{R_1}\right)\right) + A_2\left(1 - \mu\left(\frac{\rho}{R_2}\right)\right)\right] \frac{\rho - \gamma}{\theta\gamma} \quad (3.17)$$

$$\Leftrightarrow \frac{A_1}{A_2} = -\frac{\mu_2(1 + \Phi) - \Phi}{\mu_1(1 + \Phi) - \Phi}$$

where $\mu_j = \mu\left(\frac{\rho}{R_j}\right)$ for $j \in \{1, 2\}$.

It is not possible to characterize the properties of R_1 and R_2 as a function of ρ . Instead, we need now to characterize ρ as a function of R_1 and R_2 . It further complicates the multiplicity problem by perhaps increasing its dimensionality. One may have to address this numerically. Suppose a solution exists such that $R_1, R_2 > \rho > \gamma$. In equilibrium,

$$k_j = f_{k_j}^{-1}\left(\frac{R_j + \delta - 1}{z_j}\right), \text{ for } j = 1, 2 \quad (3.18)$$

and the market clearing for corporate loans requires supply of corporate loans, l_j to be

$$l_j = A_j \left[1 - \mu\left(\frac{\rho}{R_j}\right)\right] \left(1 + \frac{\rho - \gamma}{\theta\gamma}\right), \text{ for } j = 1, 2 \quad (3.19)$$

Under a closed economy, the aggregate supply of corporate loans in normal times would be the total deposit collected by the banks. However, equation (3.19) is a novel finding in our paper where the aggregate corporate lending does not only depend on the savings of the households, i.e., A , but on other factors as well, such as the corporate lending rate R , interbank lending rate ρ , and the storage components, γ and θ .

There are now five unknowns in the model: $\{R_1, R_2, k_1, k_2, \rho\}$. If an equilibrium exists, they should sensibly solve the five equations: (3.17), (3.18), and (3.19).

If there is no solution to the above problem, then we are back in the crisis world. Both countries will be in autarky with their capital employment and the loan rates given by the following two equations

$$l_j = \left[1 - \mu\left(\frac{\gamma}{R_j}\right)\right] A_j = f_{k_j}^{-1}\left(\frac{R_j + \delta - 1}{z_j}\right), \text{ for } j = 1, 2$$

while the amount stored in each country equals $\mu\left(\frac{\gamma}{R_j}\right)A_j$.

We conclude this subsection with presenting the banking sector's return on deposits. The return on deposits depends on three factors: corporate loan revenue, interbank lending revenue, and the borrowing cost. These factors are given by: corporate loan revenue $\int_{\bar{p}_j}^1 p[R_j(1 + \Phi)]d\mu(p)$, interbank loan revenue $\int_0^{\bar{p}_j} p\rho d\mu(p)$, and the interbank borrowing cost $\int_{\bar{p}_j}^1 p\rho\Phi d\mu(p)$. The banking sector's return on deposit thus is given by

$$r_j \equiv \int_0^1 r_j(p)d\mu(p) = \begin{cases} \int_{\bar{p}_j}^1 p[R_j(1 + \Phi)]d\mu(p), & \text{for equilibrium with trade} \\ R_j \left[\int_{\gamma/R_j}^1 p d\mu(p) + \frac{\gamma}{R_j} \mu\left(\frac{\gamma}{R_j}\right) \right], & \text{otherwise} \end{cases} \quad (3.20)$$

in which under the equilibrium trade, the first term represents the interbank lending revenue of a mass of $\mu(\bar{p}_j)$ banks that are lenders, and the second term denotes the corporate loan revenue plus the profit from market funded capital for a mass of $(1 - \mu(\bar{p}_j))$ banks that are borrowers. In contrast, when there is no equilibrium of trade, no banking intermediation exists and, a mass of $\mu(\gamma/R_j)$ banks find it optimal to use the storage technology.

Before we proceed to the quantitative analysis, a general equilibrium of the world economy in which two countries connected only with open interbank market is defined as follows. The state variables for a particular individual's optimization problem in country j at time t are (i) the individual asset holdings a_{jt} , (ii) the aggregate asset holdings $\{A_{1t}, A_{2t}\}$, and (iii) the realization of the technology shocks $\{z_{1t}, z_{2t}\}$. Let $\mathbf{s}_t = \{A_{1t}, A_{2t}, z_{1t}, z_{2t}\}$. In the sequel, we denote by $\Gamma_j(\mathbf{s}_t)$ the perceived law of motion of aggregate assets in the country j and by $R_j(\mathbf{s}_t)$, $r_j(\mathbf{s}_t)$, and $\rho(\mathbf{s}_t)$ and $w_j(\mathbf{s}_t)$ the pricing functions for corporate loans, deposits, interbank loans and labor, respectively; we also denote by $\pi_j(\mathbf{s}_t)$ and $\chi_j(\mathbf{s}_t)$ the profit and rebate functions. All these functions are function of aggregate assets $\{A_{1t}, A_{2t}\}$ and productivity $\{z_{1t}, z_{2t}\}$, which are both taken as given by the household and capture the presence of externalities. The household j 's recursive optimization problem writes as

$$V(a_t, \mathbf{s}_t) = \max_{a_{t+1}, c_t, n_t} u(c_t - v(n_t)) + \beta \mathbb{E}_t V(a_{t+1}, \mathbf{s}_{t+1})$$

subject to

$$a_{t+1} + c_t = r(\mathbf{s}_t)a_t + w(\mathbf{s}_t)n_t + \pi(\mathbf{s}_t) + \chi(\mathbf{s}_t)$$

$$A_{t+1} = \Gamma(\mathbf{s}_t)$$

where, in the calibrated version of the model, $u(x) = x^{1-\sigma}/(1-\sigma)$ and $v(n) = n^{1+\nu}/(1+\nu)$. The solution to this problem is a set of decision rules $a(a_{j_t}, \mathbf{s}_t)$, $n(a_{j_t}, \mathbf{s}_t)$, and $c(a_{j_t}, \mathbf{s}_t)$.

The firm's problem in country j is simply given by $\max_{k_t, h_t} z_t F(k_t, h_t) + (1-\delta)k_t - w(\mathbf{s}_t)h_t - R(\mathbf{s}_t)k_t$, which leads to the decision rules $k(\mathbf{s}_t)$ and $h(\mathbf{s}_t)$. Finally, the solution of banks' problem in country j leads to the aggregate loans $l(\mathbf{s}_t)$ and $\phi(\mathbf{s}_t) = (\rho(\mathbf{s}_t) - \gamma)/\gamma\theta$, where only aggregate assets enter the solution due to the linearity of the problem. In the recursive rational expectation equilibrium, actual and perceived law of motions coincide in each country j , respectively.

Definition 3.5.1. (*Recursive competitive general equilibrium*) A recursive competitive equilibrium of the world economy, given the aggregate state $\mathbf{s}_t = \{A_{1t}, A_{2t}, z_{1t}, z_{2t}\}$, is a sequence of prices defined by the pricing functions $R(\mathbf{s}_t)$, $r(\mathbf{s}_t)$, $\rho(\mathbf{s}_t)$ and $w(\mathbf{s}_t)$, two perceived law of motions for aggregate assets $\Gamma_1(\mathbf{s}_t)$ and $\Gamma_2(\mathbf{s}_t)$, and a set of decision rules for each country $j = 1, 2$ of $\{c(a_{j_t}, \mathbf{s}_t), a'(a_{j_t}, \mathbf{s}_t), k(a_{j_t}, \mathbf{s}_t), h(a_{j_t}, \mathbf{s}_t), \phi(a_{j_t}, \mathbf{s}_t), l(a_{j_t}, \mathbf{s}_t)\}$ with the value function $V(a_{j_t}, \mathbf{s}_t)$ such that,

1. $\{c(a_{j_t}, \mathbf{s}_t), a'(a_{j_t}, \mathbf{s}_t), n(a_{j_t}, \mathbf{s}_t)\}$ and $V(a_{j_t}, \mathbf{s}_t)$ solve the household j 's recursive optimization problem taking $R_j(\mathbf{s}_t)$, $r_j(\mathbf{s}_t)$, $w_j(\mathbf{s}_t)$, $\pi_j(\mathbf{s}_t)$, $\chi_j(\mathbf{s}_t)$ and $\Gamma_j(\mathbf{s}_t)$ as given.
2. $\{k_j(\mathbf{s}_t), h_j(\mathbf{s}_t)\}$ solve the firm j 's optimization problem taking $R_j(\mathbf{s}_t)$, $w_j(\mathbf{s}_t)$ as given.
3. $\phi_j(\mathbf{s}_t)$ solves the bank's optimization problem taking $R_j(\mathbf{s}_t)$, $r_j(\mathbf{s}_t)$ and $\rho(\mathbf{s}_t)$ as given. Aggregate loans in each country j are, respectively

$$l_j(\mathbf{s}_t) = \begin{cases} A_{j_t} \left[1 - \mu \left(\frac{\rho}{R_j(\mathbf{s}_t)} \right) \right] \left(1 + \frac{\rho - \gamma}{\theta \gamma} \right), & \text{if } A_t \leq \bar{A}(z_{1t}, z_{2t}) \\ \left(1 - \mu \left(\frac{\gamma}{R_j(\mathbf{s}_t)} \right) \right) A_{j_t}, & \text{otherwise} \end{cases}$$

4. The perceived law of motion for aggregate assets is consistent with the actual law of motions:

$$a'_j(a_t, \mathbf{s}_t) = \Gamma_j(\mathbf{s}_t).$$

5. Wages satisfy $w_j(\mathbf{s}_t) = z_{jt}F_h(k_j(\mathbf{s}_t), h_j(\mathbf{s}_t))$, and the corporate loan rate satisfies $R_j = z_{jt}F_k(k_j(\mathbf{s}_t), h_j(\mathbf{s}_t)) + 1 - \delta$, the deposit rate satisfies

$$r_j(\mathbf{s}_t) = \begin{cases} \int_{\bar{p}_j(\mathbf{s}_t)}^1 p[R_j(\mathbf{s}_t)(1 + \Phi(z_{1t}, z_{2t}))]d\mu(p), & \text{if } A_t \leq \bar{A}(z_{1t}, z_{2t}) \\ R_j(\mathbf{s}_t) \left[\int_{\frac{\gamma}{R_j(\mathbf{s}_t)}}^1 pd\mu(p) + \frac{\gamma}{R_j(\mathbf{s}_t)}\mu\left(\frac{\gamma}{R_j(\mathbf{s}_t)}\right) \right], & \text{otherwise} \end{cases}$$

where $\bar{p}_j(\mathbf{s}_t) \equiv \rho(\mathbf{s}_t)/R_j(\mathbf{s}_t)$

6. The aggregate intermediation cost rebated to the household is given by $\chi_j(\mathbf{s}_t) = (R_j(\mathbf{s}_t) - r_j(\mathbf{s}_t))A_{jt} - (R_j(\mathbf{s}_t) - \gamma)(A_{jt} - k_j(\mathbf{s}_t))$, and the firm j 's profits are equal to zero, $\pi_j(\mathbf{s}_t) = 0$.

7. Goods, labor, capital and interbank markets clear:

$$c_j(A_{jt}, \mathbf{s}_t) + a'_j(A_{jt}, \mathbf{s}_t) = z_j F(k_j(\mathbf{s}_t), h_j(\mathbf{s}_t)) + (\gamma + \delta - 1)(A_{jt} - k_{jt}) + (1 - \delta)A_{jt}$$

$$h_j(\mathbf{s}_t) = n_j(A_{jt}, \mathbf{s}_t)$$

$$k_j(\mathbf{s}_t) = l_j(\mathbf{s}_t)$$

$$A_{1t}\mu\left(\frac{\rho(\mathbf{s}_t)}{R_1(\mathbf{s}_t)}\right) + A_{2t}\mu\left(\frac{\rho(\mathbf{s}_t)}{R_2(\mathbf{s}_t)}\right) = \left[A_{1t}\left(1 - \mu\left(\frac{\rho(\mathbf{s}_t)}{R_1(\mathbf{s}_t)}\right)\right) + A_{2t}\left(1 - \mu\left(\frac{\rho(\mathbf{s}_t)}{R_2(\mathbf{s}_t)}\right)\right) \right] \frac{\rho(\mathbf{s}_t) - \gamma}{\gamma\theta}$$

8. The banking sector's absorption capacity is given by

$$\bar{A} = \bar{A}_1(z_{1t}) + \bar{A}_2(z_{2t})$$

where

$$\bar{A}_j(z_{jt}) = \left(\frac{1 - \alpha}{\vartheta}\right)^{\frac{1}{\nu}} \left(\frac{\alpha}{\bar{R} + \delta - 1}\right)^{\frac{\nu + \alpha}{\nu(1 - \alpha)}} z_{jt}^{\frac{1 + \nu}{\nu(1 - \alpha)}}$$

3.6 Quantitative Analysis

Following Boissay et al. (2016), we investigate the quantitative properties of the model using the calibrated values of parameters presented in Table 3.1.

For the Foreign country, we use the AR(1) estimates that are consistent with those of the peripheral European countries so that $\rho_z = 0.89$ and $\sigma_{zf} = 3$ percent. We solve the model numerically

Table 3.1 Calibration

Parameter	Value	Description
β	0.971	Discount factor (Deflated for growth)
ν	0.500	Inverse of Frisch elasticity
ϑ	0.945	Labor disutility
σ	2.000	Risk aversion
α	0.300	Capital elasticity
δ	0.100	Capital depreciation rate
ψ	1.012	Growth factor
σ_{zh}	0.013	Std. of productivity shock in Home country
σ_{zf}	0.030	Std. of productivity shock in Foreign country
ρ_z	0.890	Persistence of productivity shock in both Home and Foreign country
λ	26.000	Bank distribution: $\mu(p) = p^\lambda$
θ	0.085	Diversion cost
γ	0.952	Storage technology

Table 3.2 Model Statistics on Recessions

	Financial	Other	All	Severe	Mild
Events	644	4708	5352	1784	1784
Frequency(%)	1.29	9.42	10.70	3.57	3.57
Duration(years)	2.16	1.34	1.43	1.98	1.04
Output Loss(%)	-10.79	-3.31	-4.21	-7.38	-2.13

using the collocation method, with decision rules iterations as in Boissay et al. (2016), allowing for discontinuities in the asset accumulation at the points at which the economy switches regime.

We first analyze the response of the Home country to a positive one standard deviation productivity shock, when the economy is initially at the steady state associated with the long-run average of z_t . Figure 3.1 compares the dynamics in the model (thick plain line) with those in a frictionless model (round markers). The responses of output, hours worked, consumption, and investment are relatively larger than those in Boissay et al. (2016). The intuitive explanation is that due to a larger size interbank market, our two-country model exhibits much more amplification than the closed-economy model as well as the frictionless RBC model.

Figure 3.2 and 3.3 report the typical path to financial recessions of the Home country and Foreign country, respectively. These are depicted from the simulation of the model over 50,000

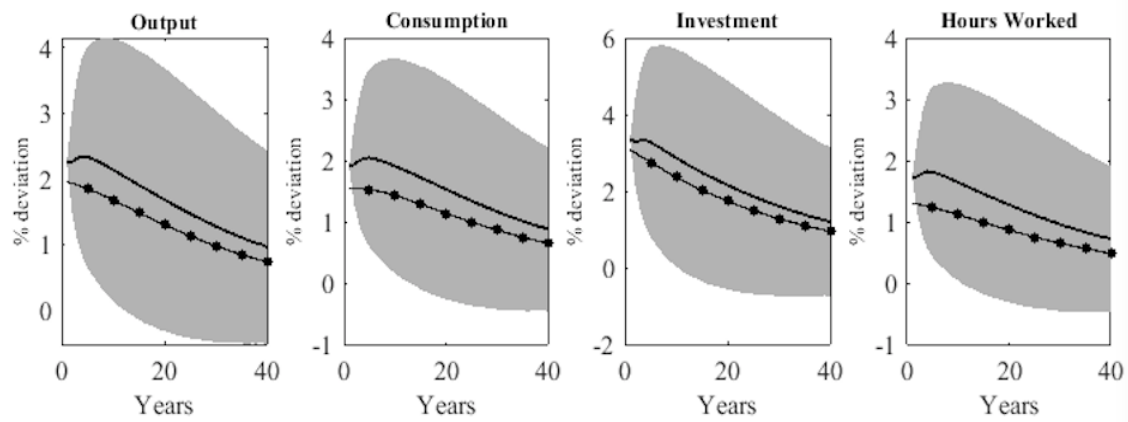


Figure 3.1 Impulse response to a one standard deviation technology shock

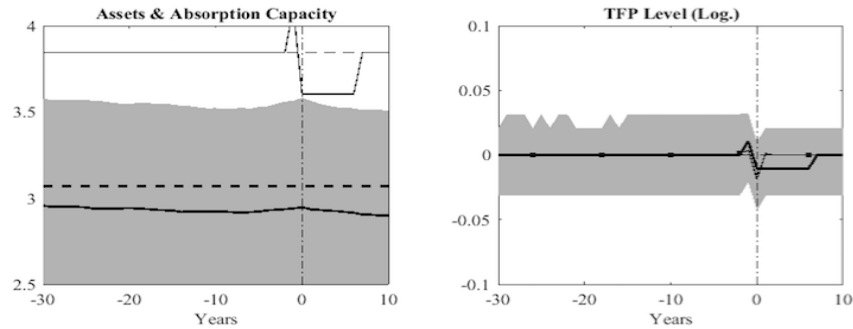


Figure 3.2 Typical path to financial recessions in the Home country

periods with the years of recessions and crises identified. By convention, period 0 corresponds to the period when the financial recession (or the crises) bursts. Note that in the simulation, only the Foreign country is hit by the shock so that its banks' absorption capacity (thick plain line) as well as the endogenous dynamics of banks' assets (thin plain line) cross out with each other around the crises, while those in Home country are relatively stable.

Since two countries share the interbank market, the Home country experiences financial recessions due to the contagion effect. Figure 3.4 shows the dynamics of the output and credit gaps around recessions in the Home country.

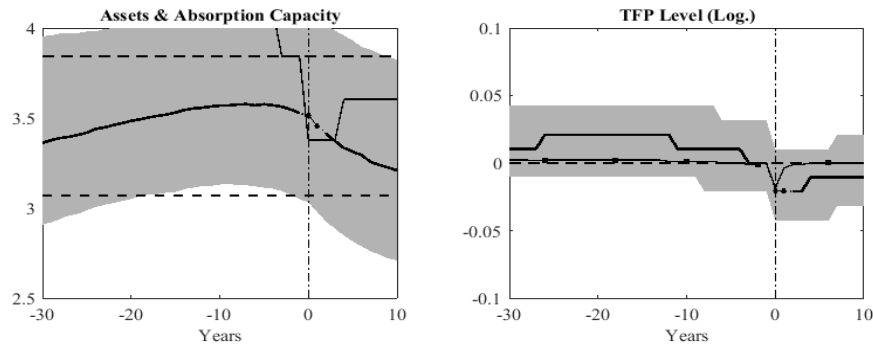


Figure 3.3 Typical path to financial recessions in the Foreign country

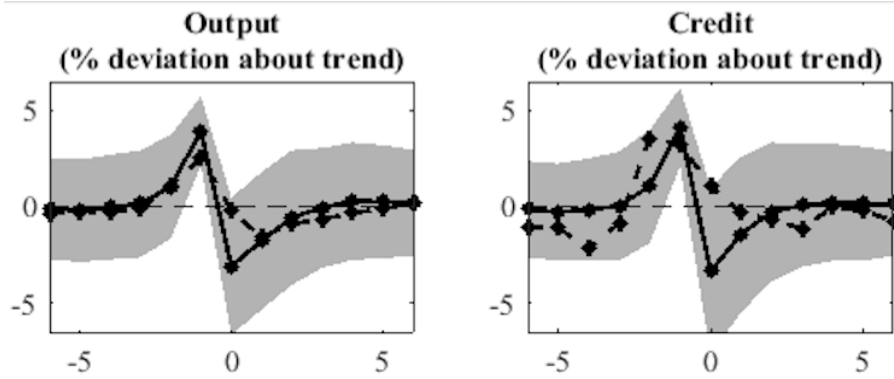


Figure 3.4 Dynamics of output and credit gaps around recessions in Home country

Compared with that in the closed economy, we see that the relative size and duration of financial recessions from simulations both increase in the world economy. Thus, they confirm that the model generates deeper and longer financial recessions than other recessions, which are associated with a boom-bust cycle in the credit. However, our two-country model generates financial recessions with a frequency of 1.29 percent, nearly 50 percent lower than that in a closed economy (See Boissay et. al. 2016, page 519). In other words, the very nature of a shared interbank market makes the economy less susceptible to negative productivity shocks, but the resulted crisis are more pronounced if they occur. The intuition is that, an internationally integrated interbank market endowed with a relatively larger "absorbing capacity" acts as a shield against negative shocks and fluctuations, which makes crisis less frequent. Rationally anticipating this, borrowers have more

incentive to leverage and the gain from diversion increases as well. The interbank moral hazard issue potentially becomes more severe and implies larger macroeconomic contractions once induced by a crisis.

3.7 Concluding Remarks

In this paper, we offered a two-country model of banking crises. Banks are heterogeneous with respect to their private intermediation efficiency. This heterogeneity gives rise to an interbank market. However, due to the moral hazard problem along with private intermediation skills, less efficient banks may have an incentive to borrow in the interbank market and divert funds at a lower return asset. A borrowing capacity constraint restricts these less efficient banks from diverting the funds. However, a lower corporate lending rate and interbank rate increases the incentive of these banks to divert funds and the interbank market responds by lowering the borrowing capacity. At some point, the interbank market goes to a freeze if the rates of return are sufficiently low.

A sequence of favorable productivity shocks lead to an expansion of credit. However, when the productivity growth goes down from an increasing trend, corporate lending rate goes down along with the demand. Counterparty risk goes up and the market stops lending to each other. When two countries are integrated through interbank lending, a market freeze in one country leads to a freeze in another country as well. Either two countries have an operating interbank market or both go into a freeze. Excessive credit creation by one country triggers a two-country wide credit freeze. The countries may also go into a freeze from over low productivity in either of the two countries.

In this paper, we made some strong assumptions for tractability reasons and presented a stylized model. Indeed, we left out some features of the banking sector that would deserve attention for future extensions. For example, we assume that households cannot lend the firms directly and firms cannot issue any equity or debt securities. Moreover, we assume firms do not default on their loans. Another possibility would be time-varying return on the storage technology. All these extensions are left for future research.

3.8 References

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CHAPTER 4. EFFICIENT FRICTIONS: FROM CREDIT TO LABOR MARKET

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4.1 Abstract

We introduce ex-ante heterogeneous skilled workers and endogenous labor market participation a la Albrecht et al. (2010) into a single-period version of credit and labor market search model as in Wasmer and Weil (2004). It is shown that Hosios condition fails to achieve constrained efficiency since the resulting labor market is too tight, and credit market frictions can be welfare improving. In particular, we consider two cases in which workers effectively share the vacancy cost or not, respectively. In the former case, wage contract is settled after loan contract and workers have to share the vacancy cost, which helps restore constrained efficiency at Hosios rule given a sufficiently but not too much tight labor market that is induced by appropriate credit frictions. In the latter case, workers get paid a constant share of the output and the presence of a frictional credit market can be welfare improving. In both cases, perfect smooth credit is not preferred, and credit market freeze is always reducing welfare.

4.2 Introduction

It has been well-documented in literature that credit market imperfections give rise to high unemployment and low output and amplify macroeconomic volatility through a financial accelerator, see, e.g. Acemoglu (2011), Wasmer and Weil (2004). Recent financial crisis and the induced slow recovery of labor market in United States also highlight the importance of an integrated approach by incorporating both credit and labor market frictions in a unified framework ¹ to analyze the effects of a credit crunch and its implications, see, for example, Buera et al. (2015).

In a recent paper, Albrecht et al. (2010) (hereafter, ANV) shows in a simple labor search model that Hosios rule, as in Hosios (1990), fails to yield constrained efficiency. In their model, heterogeneous skilled workers choose to either participate in market production or engage in home production. When Hosios condition holds, there are excess entries of workers due to too many vacancies posted in equilibrium. In other words, labor market is too tight. The source of this inefficiency, as suggested by ANV themselves, is that Hosios condition only internalize search externalities while have the average productivity effect unaccommodated, i.e. the increased entry of marginally less productive workers contributes to a lower equilibrium level of expected market output and thus reduces social welfare.

Instead of pursuing a somewhat generalized Hosios rule (Julien and Mangin, 2017), we take a different attempt to restore efficiency by considering a more general setup that features real world frictions pertaining to labor market. Indeed, in ANV, one of the reasons that there are too many vacancies posted in equilibrium is because it is implicitly assumed that entrepreneurs find no frictions to pay for the vacancy cost so as to recruit workers. In real world, however, it is usually the case in which firms search for and bargain with bankers or investors in order to finance the cost of opening a job vacancy, due to credit market imperfections even at normal times. Hence, we decide to tackle the problem by bringing back to the center stage a credit market in a fashion as in the seminal work by Wasmer and Weil (2004). The basic insight is that by lowering labor market tightness through a general-equilibrium interaction, credit market imperfections can be welfare

¹See Petrosky-Nadeau and Wasmer (2017) for a formal textbook treatment.

improving. In particular, in equilibrium, entrepreneurs now find it difficult to get matched with bankers due to search and matching frictions in the credit market. As a result, less vacancies can be created and only sufficiently productive workers choose to participate in market production.

We consider two different cases as for how market wage is determined in equilibrium since it directly affects workers decision-making of entry. In the first case, labor search comes before credit search (see fn.3) and as a result, workers get paid a constant share of their output. An immediate implication of this arrangement is that the marginal workers productivity as well as the average market productivity are both decreasing in the job market tightness. In other words, a relatively tighter labor market is always preferred from the workers perspective so that it attracts marginally less productive ones to enter, which leads to inefficiency, as in ANV. Thus, the adding of a frictional credit market does not necessarily help the economy restore its constrained efficiency at Hosios rule, since it changes nothing but only to increase the effective vacancy cost that falls solely on the entrepreneurs burden. However, we find that the economy with credit market frictions can be more desirable than the one with perfect smooth credit. By lowering the equilibrium labor market tightness, credit market imperfections can improve welfare. We also show that credit market freeze in which firms are unable to get any financing at all is always undesirable as in equilibrium no vacancy could be opened on the market and all workers get stuck at home production.

In the second case, we emphasize the potent role of credit frictions by letting loan contract settle before wage contract. Since banker enjoys a first-mover advantage in this sequential bargaining game, workers have to share the vacancy cost in contrast to the previous case. As a result, there emerges a threshold of labor market tightness beyond which the cut-off entry productivity and average market productivity increase as labor market becomes more tight. This is because even though it is more likely to get paired with an entrepreneur in a tighter job market, the wage payoff is nevertheless decreasing. It is shown that constrained efficiency can be restored at Hosios rule in a sufficiently but not too much tight labor market induced by appropriate credit market frictions. This additional outcome is delivered exactly through the common sharing of vacancy cost for all market participants which drives the internalization of individual workers output effect upon entry.

That is, marginally less productive workers are deterred from entry as they rationally choose to avoid the un-affordable expected vacancy cost. Comparing to the first case, it turns out that which market search comes first matters for the efficiency.

To our knowledge, this paper is the first to address the welfare improving role of credit search and matching frictions in the labor search framework. This surprising result contrast sharply to the conventional wisdom which suggests that credit market imperfections generally be associated with welfare losses, and that a perfectly smooth credit market will tend to allocate resources more efficiently. On the contrary, we find that certain market frictions can be welfare enhancing, and may also restore the constrained efficiency at Hosios rule under appropriate market search sequence. However, the improved efficiency comes at the cost of higher equilibrium labor market unemployment and lower output.

4.2.1 Related Literature

Some papers in recent literature have also extended ANVs basic environment. Examples include Gavrel (2011), Charlot et al. (2013), Masters (2015), and Julien and Mangin (2017). In particular, Gavrel (2011) shows that if the social planner is allowed to maximize the welfare with respect to both job creation (i.e. $\frac{\partial W}{\partial \theta}$) and market participation (i.e. $\frac{\partial W}{\partial y^*}$), the resulted market participation would be too high, while the labor market tightness is actually partially efficient. He then finds that subsidizing non-participants could improve market efficiency and reduce unemployment. This prescription is based on the implicit assumption that public policy can influence the participation decisions of workers. On the contrary, our paper shows that market efficiency can be enhanced in a laissez-faire regime through the introduction of a credit market with efficient frictions. Julien and Mangin (2017) have also managed to restore efficiency by proposing a modified version of Hosios condition that captures not only the standard search externalities but also the dampening effect of marginal workers on the average market productivity upon entry. An important difference between these papers and ours is that they all choose to directly tackle the issue, while in our model, the interaction between the credit and labor markets largely alters both entrepreneurs and workers

decision making of entry, and emphasizes the importance of incorporating credit frictions to the traditional labor search model in terms of welfare analysis.

Several recent papers exploit the idea that asset market frictions can be welfare improving, see for example Berentsen et al. (2014) and Geromichalos and Herrenbrueck (2016). In these papers, agents fail to internalize the pecuniary externality in the portfolio choice when trading at the secondary asset market. Berentsen et al. (2014) argues that the optimal policy response is to restrict full access to the financial market. Our paper shares a similar spirit to that of Geromichalos and Herrenbrueck (2016), who find that the equilibrium welfare is always higher in the model where the secondary asset market is a frictional over-the-counter (OTC) market, if the inflation is not too high.

In a closely-related paper, Petrosky-Nadeau (2013) examines the effect of a credit crunch on the total factor productivity (TFP) in a dynamic general equilibrium model featured with heterogeneity in firm productivity and endogenous job creation and destruction. He shows that adverse shocks to credit market destroy those least productive jobs and thus raise both aggregate TFP and unemployment. There are two main differences between Petrosky-Nadeau (2013) and our paper. First, he does not characterize the equilibrium efficiency, while we do, though at a static setting. Second, we show that at normal times a frictional credit market can improve social welfare and is more desirable than perfect smooth credit, though a complete credit market freeze always reduces social welfare. Thus, we consider our work as a complement to his.

Finally, Petrosky-Nadeau and Wasmer (2013) extends the credit and labor search model in Wasmer and Weil (2004) to show that financial frictions add an additional entry cost to job creation costs and create labor markets cyclical volatility. They also find that the elasticity of labor market tightness to productivity shocks is increasing in total financial costs, and can be minimized at the credit-market Hosios condition.

The remainder of the paper is organized as follows. In Section 2 we present the model and show the main results of the paper. Section 3 concludes and suggests future work.

4.3 The Model

The basic environment is a single period version of the economy as in Wasmer and Weil (2004). There are three types of agents in this economy: a continuum of entrepreneurs who cannot operate alone and need funds to start up the business, a continuum of bankers that have sufficient resources to finance the project, and a measure 1 of workers that can produce and realize the entrepreneurs ideas into output.

The timeline of events is as follows. In the first stage, an entrepreneur goes and finds a banker who agrees to pay for the vacancy cost γ to recruit a worker. The search costs for the entrepreneur and the banker are c and k , respectively. Notice that search costs are non-pecuniary and can be interpreted as time and effort spent on it. The probability that the entrepreneur meets with a banker is given by ²

$$\frac{M_C(E, B)}{E} = M_C(1, \phi^{-1}) = p(\phi)$$

where $\phi = E/B$ is defined as credit market tightness from the point of view of firms, given as the ratio of the number of entrepreneurs E to the number of bankers B that are searching in this market. Thus the probability that a banker meets with an entrepreneur is $M_C(E, B)/B = \phi \cdot p(\phi)$ and we have $\frac{dp(\phi)}{d\phi} < 0$, and $\frac{d[\phi \cdot p(\phi)]}{d\phi} > 0$.

In the second stage, the successfully matched entrepreneur goes to the labor market to recruit a worker. The corresponding matching probability is given by

$$\frac{M_L(v, u)}{v} = M_L(1, \theta^{-1}) = q(\theta)$$

in which $\theta = v/u$ is labor market tightness and similarly we have $q'(\theta) < 0$, and $\frac{d[\theta \cdot q(\theta)]}{d\theta} > 0$. The worker does not incur any search cost but need to choose between whether to participate in the labor market or not in the first place. In particular, she will get a payoff of z (leisure or payoff

²As in Pissarides (2000), the matching function $M_C(E, B)$ (so is $M_L(v, u)$, see below) is assumed increasing in both its arguments, concave, and homogeneous of degree 1 (or constant returns to scale); e.g. Cobb-Douglas form $M_C(E, B) = \chi_C E^{1-\epsilon} B^\epsilon$ and $M_L(v, u) = \chi_L v^{1-\eta} u^\eta$ where χ_C and χ_L are scale parameters. This is one of the simplest way in catching matching frictions. In the basic setup, as pointed by ANV, this also suggests that all workers have an equal chance of finding a job, regardless of their individual productivity.

from home production) with certainty if she chooses to stay out of the labor market. Otherwise, she will get a payoff based on her individual market productivity y , the bargaining protocol (see details below) over how to split the output, and labor market tightness. Workers productivity y is a random draw from a continuous cumulative distribution function $F(y)$ with the support $[0, \bar{y}]$ and is *i.i.d.* across workers. If she chooses to search for an entrepreneur, the matching probability is $\theta q(\theta)$, and if she fails to get matched, the payoff would be zero. Apparently, there exists a cutoff value of y^* above which the worker finds a higher expected payoff of participating in the labor market.

In the third stage, the output is split according to some (predetermined) bargaining protocols among the entrepreneur, the banker, and the worker. We consider two cases here. The first case is that the worker will get a fraction α of the output y that she produces, as in ANV. Then the rest $(1 - \alpha)y$ will be split between the entrepreneur and the banker according to Nash bargaining. One way to think about this case is that when negotiating, the worker is actually facing with a joint venture of the entrepreneur and the banker. The asymmetric treatment ³ of the bargaining positions of the three agents is a mere reflection of the fact that the search as well as the vacancy cost incurred by the venture entity are irreversible and workers are the only productive ones in the economy.

The second case is the sequential bargaining game as in Wasmer and Weil (2004): the entrepreneur first ⁴ negotiates with the banker over the loan contract in stage 1, and then bargains with the worker who decides to participate in the labor market over the wage contract in stage 2. Thus, we need to solve this game backward and as will be clear later, this would have a very different implication for the workers entry decision.

³Another way to formalize this is to put labor search first and then credit search, and γ should instead be interpreted as, say, a fixed capital cost financed through bankers. The key is that worker does not effectively share γ .

⁴This would be a more realistic structure for those start-up firms. In this case, banker issues debt first and thus is more senior than the residual claimants, i.e. entrepreneur and worker.

Before we move to solve the model, let $\hat{y} = \int_{y^*}^{\bar{y}} \frac{y}{1-F(y^*)} dF(y)$ denote the average market productivity of workers, w the wage rate, ρ the payoff to the banker, and $(y - w - \rho)$ the firm's profit⁵.

4.3.1 Case I: worker gets paid of $\alpha \cdot y$

In this case, a worker participates in the labor market if and only if $\alpha y \cdot \theta q(\theta) \geq z$, that is,

$$y \geq y^* = \frac{z}{\alpha \theta q(\theta)} \equiv h(\theta) \quad (4.1)$$

note that $h'(\theta) < 0$, i.e., the marginal worker's productivity decreases as labor market becomes more tight. The labor market tightness is, by definition

$$\theta = \frac{v}{1 - F(y^*)} = \frac{v}{1 - F(h(\theta))}$$

where

$$v = M_C(E, B) = Ep(\phi)$$

since only when the entrepreneur and the banker get matched can the vacancy be posted through the financing from the latter. Differentiating with respect to E (but ϕ is fixed, see below), we get

$$\begin{aligned} \frac{d\theta}{dE} &= \frac{\partial \theta}{\partial E} + \frac{\partial \theta}{\partial y^*} h'(\theta) \frac{dy^*}{dE} \\ &= \frac{p(\phi)}{1 - F(y^*)} + \frac{\theta f(y^*)}{1 - F(y^*)} h'(\theta) \frac{d\theta}{dE} \end{aligned} \quad (4.2)$$

so $\frac{d\theta}{dE} > 0$.

Banker and entrepreneur share the surplus of their joint venture according to the generalized Nash bargaining rule

$$\rho = \arg \max \left(\rho - \frac{\gamma}{q(\theta)} \right)^\beta \left((1 - \alpha) \hat{y} - \rho \right)^{1-\beta}$$

⁵We will hereafter refer interchangeably to entrepreneurs or firms.

where $\beta \in (0, 1)$ measures the banker's bargaining power within the venture, and $(1 - \alpha)\hat{y}$ indicated that sharing rule is of the *expected* leftover. The expected repayment to the banker is thus

$$\rho = \beta \cdot (1 - \alpha)\hat{y} + (1 - \beta)\frac{\gamma}{q(\theta)} \quad (4.3)$$

which is essentially a weighted average the expected total surplus of the venture and (compensation for) the effective vacancy cost.

In equilibrium, free entry of bankers and entrepreneurs on the credit and labor market ensures that

$$-k + \phi p(\phi)[- \gamma + \rho \cdot q(\theta)] = 0$$

and

$$-c + p(\phi) \cdot q(\theta) \cdot [(1 - \alpha)\hat{y} - \rho] = 0$$

rearrange the above two equations and combine with equation (4.3), we get

$$\frac{k}{\phi p(\phi)q(\theta)} = \beta \left((1 - \alpha)\hat{y} - \frac{\gamma}{q(\theta)} \right) \quad (4.4)$$

$$\frac{c}{p(\phi)q(\theta)} = (1 - \beta) \left((1 - \alpha)\hat{y} - \frac{\gamma}{q(\theta)} \right) \quad (4.5)$$

in which the left-hand side are the effective search costs while the right-hand side are the corresponding benefits. Note that the equilibrium credit market tightness is given by taking the ratio of (4.4) and (4.5)

$$\phi^* = \frac{1 - \beta}{\beta} \cdot \frac{k}{c}$$

which depends only on the features of the credit market. Also in the limit, as credit matching becomes frictionless (i.e., $p(\phi) \rightarrow +\infty$ for all ϕ), equilibrium labor market tightness $\theta \rightarrow \bar{\theta}$, where $\gamma/q(\bar{\theta}) = (1 - \alpha)\hat{y}$. We thus have the following lemma ⁶

⁶This is basically a variant of Corollary 1 from Wasmer and Weil (2004).

Lemma 4.3.1.1. *The average market productivity \hat{y} decreases with labor market tightness θ . As a result, credit market imperfections lower equilibrium labor market tightness: $\theta^* < \bar{\theta}$.*

Proof. First observe that $\hat{y} = \int_{y^*}^{\bar{y}} \frac{y}{1-F(y^*)} dF(y)$ in which $y^* \equiv h(\theta)$ and $h'(\cdot) < 0$, hence

$$\begin{aligned} \frac{d\hat{y}}{d\theta} &= \frac{f(y^*)}{[1-F(y^*)]^2} \cdot h'(\theta) \int_{y^*}^{\bar{y}} yf(y)dy - \frac{y^*f(y^*)}{1-F(y^*)} \cdot h'(\theta) \\ &= \frac{f(y^*)h'(\theta)}{1-F(y^*)} \int_{y^*}^{\bar{y}} \frac{(y-y^*)f(y)}{1-F(y^*)} dy < 0 \end{aligned}$$

To show $\theta^* < \bar{\theta}$, we prove by contradiction. Suppose instead $\theta^* > \bar{\theta}$, then $\hat{y}(\theta^*) < \hat{y}(\bar{\theta})$ and $q(\theta^*) < q(\bar{\theta})$. However, from equation (4.5) we know this cannot be true since

$$\begin{aligned} \frac{\gamma}{q(\theta^*)} &= (1-\alpha)\hat{y}(\theta^*) - \frac{c}{1-\beta} \cdot [q(\theta^*)]^{-1} \cdot \left[p \left(\frac{1-\beta}{\beta} \cdot \frac{k}{c} \right) \right]^{-1} \\ &< (1-\alpha)\hat{y}(\bar{\theta}) \\ &= \frac{\gamma}{q(\bar{\theta})} \end{aligned}$$

a contradiction. Therefore, it must be that $\theta^* < \bar{\theta}$ □

Now we can characterize efficiency. The social planner is trying to choose the number of entrepreneurs ⁷ E to maximize

$$W = zF(y^*) + \theta q(\theta)(1-F(y^*)) \int_{y^*}^{\bar{y}} \frac{yf(y)}{1-F(y^*)} dy - \gamma v - kB - cE \quad (4.6)$$

where the social welfare is the sum of home production and market production, net of the following cost in total: the vacancy cost, and the search cost in the credit market incurred from both bankers and entrepreneurs. Directly taking derivatives gives

⁷Once E is determined, the number of bankers, B , can be calculated from that $v = M_C(E, B)$ since $\theta = \frac{v}{1-F(h(\theta))}$ where θ is pinning down from equation (4.4) and (4.5).

$$\begin{aligned}
\frac{dW}{dE} &= z f(y^*) h'(\theta) \frac{d\theta}{dE} + [q(\theta) + \theta q'(\theta)] \frac{d\theta}{dE} \int_{y^*}^{\bar{y}} y f(y) dy - \theta q(\theta) y^* f(y^*) h'(\theta) \frac{d\theta}{dE} - \gamma p(\phi) - k/\phi - c \\
&= z - y^* \theta q(\theta) + \hat{y} \theta [q(\theta) + \theta q'(\theta)] f(y^*) h'(\theta) \frac{d\theta}{dE} + [q(\theta) + \theta q'(\theta)] p(\phi) \hat{y} - \gamma p(\phi) - k/\phi - c \\
&= -(1 - \alpha) y^* \theta q(\theta) + \hat{y} \theta (1 - \alpha) q(\theta) f(y^*) h'(\theta) \frac{d\theta}{dE} + q(\theta) p(\phi) \left((1 - \alpha) \hat{y} - \frac{\gamma}{q(\theta)} - \frac{k}{\phi p(\phi) q(\theta)} - \frac{c}{p(\phi) q(\theta)} \right) \\
&= (1 - \alpha) \theta q(\theta) \hat{y} f(y^*) h'(\theta) \frac{d\theta}{dE} \\
&< 0
\end{aligned} \tag{4.7}$$

where we evaluate dW/dE at Hosios condition $\alpha = -\theta q'(\theta)/q(\theta)$, replace the second $d\theta/dE$ in the first equality with (4.2), and use free entry conditions (4.4) and (4.5) to yield the last equality above.

When Hosios condition holds, in equilibrium there are too many entries of entrepreneurs, which give rise to too many vacancies posted (since $\phi^* = E/B$ is fixed and $v = M_C(E, B)$). As a result, labor market tightness is too high. Thus, our finding is robust and consistent with that of ANV. This is because in our setting, the introduction of a frictional credit market changes nothing but only to indirectly increases the vacancy cost for the entrepreneur. This can be easily revealed by rearranging equation (4.5) so that the "effective" vacancy cost for an entrepreneur is

$$\gamma^{\text{effective}} = \frac{c}{(1 - \beta)p(\phi)} + \gamma \tag{4.8}$$

if otherwise only labor search is considered in the analysis, as in ANV. From the social planners perspective, choosing an optimal number of entrepreneurs E is equivalent to setting an optimal level of vacancy cost γ . In the Appendix, *however*, we show that if the government can freely choose γ , which can be rationalized either as a tax or license fee charged in practice, in equilibrium there are insufficient vacancies created and the labor market tightness is too low at Hosios rule.

Back to the current setup, we already know that $\theta^* < \bar{\theta}$, which means credit search and matching frictions might improve social welfare by lowering equilibrium labor market tightness as compared to the case of perfect smooth credit. This is done by deterring the entry of less productive workers

through the general-equilibrium interactions between credit and labor markets, which contrasts sharply to the conventional wisdom that in general credit frictions only lead to lower social welfare.

However, we remind our readers with some *caveats* to the above conclusion. In fact, as $p(\phi) \rightarrow 0$, $d\theta/dE \rightarrow 0$ (see equation (4.2)) and $dW/dE \rightarrow 0$, and we get $E = 0$ and $W = z$. That is, when credit matching becomes impossible, no entrepreneurs enter and as a result, all workers choose to disengage in market production. In this sense, a severe financial crises like the one in 2008 that cause an almost complete credit market freeze would heavily reduce the average market productivity, and lead to high equilibrium unemployment and low output in the economy (see Petrosky-Nadeau, 2013).

More formally, we have the following proposition

Proposition 4.3.1.1. *When Hosios condition holds in the labor market in which workers get paid a constant share of their output, there are excess entries of entrepreneurs and the equilibrium labor market tightness is too high. Search and matching frictions in the credit market can be welfare improving by deterring the entry of less productive workers via a less tight labor market. However, perfect smooth credit is never desirable and credit market freeze always leads to welfare losses.*

Proof. See above. □

Example 4.3.1.1. *We assume that $F(\cdot)$ follows Pareto distribution as in Petrosky-Nadeau (2013) and adopt the following parameters' values as shown in Table 4.1. Then we compute the social welfare in two cases: one in which $p(\phi) = 1$, the other where $p(\phi) \rightarrow \infty$, i.e., frictionless credit market from the perspective of entrepreneurs and thus $v = E$. Table 4.2 presents the results.*⁸

Indeed, as pointed out by Julien and Mangin (2017), the standard Hosios condition only internalizes search externalities but fails to capture the so-called *output externality* and *participation externality* in the model of heterogeneous workers with respect to productivity, see, also, Julien

⁸The average market productivity is thus given by

$$\begin{aligned}\hat{y} &= \int_{y^*}^{\bar{y}} \frac{y}{1 - F(y^*)} \mu m^\mu y^{-\mu} dy \\ &= \frac{\mu}{1 - \mu} m^\mu \frac{1}{1 - F(y^*)} \left((\bar{y})^{1-\mu} - (y^*)^{1-\mu} \right)\end{aligned}$$

Table 4.1 Summary of baseline parameter values

Scale m / curv. μ	0.1 / 1.1	\bar{y}	2
Credit matching level χ_C	7	Labor matching level χ_L	0.66
elasticity ϵ	0.5	elasticity η	0.72
bank barg. power β	0.2	worker barg. power α	0.72
Credit-market search		Labor-market search	
Bank cost k	0.003	vacancy cost γ	0.13
Entrep. cost c	0.003	home production z	0.32

Table 4.2 Social welfare w./ credit frictions and w./o it

	W	E	v	\hat{y}	θ
frictionless credit mkt.	0.3262	0.1414	0.1414	0.7607	1.1147
frictional credit mkt.	0.3263	0.1359	0.1359	0.7617	1.0812

and Mangin (2016). Moreover, as we have assumed workers being in a relatively condescending bargaining position compared to the joint venture of entrepreneurs and bankers ⁹, the former do not share the cost of posting a vacancy, which also adds to a layer of excess entry and thus worsens the equilibrium allocation. Credit market frictions can only mitigate but not fully eliminate this source of inefficiency.

What happens if the wage contract comes after the loan contract is determined? In the sequential bargaining case discussed below, as we would expect, now workers have to take into account the vacancy cost since bankers enjoy the first mover advantage when negotiating with entrepreneurs. Moreover, credit market frictions improve social welfare only when the induced labor market is sufficiently tight, with which it restores constrained efficiency at Hosios rule.

⁹This assumption can be rationalized either as a result from the collective bargaining of labor union, or simply as a legal requirement by government. For example, firms in United States in need of recruiting high-tech immigrant workers are supposed to sponsor their employees working visa application as well as to pay a prevailing wage in that position, according to the Immigration and Nationality Act, section 101(a)(15)(H).

4.3.2 Case II: a sequential bargaining game

In this case, we solve the model from backward by first characterizing the wage bargaining between workers and entrepreneurs, then the loan bargaining between entrepreneurs and bankers, and finally pinning down the workers entry decisions.

Workers and entrepreneur share the surplus according to the generalized Nash bargaining rule by taking ρ as given

$$w = \arg \max w^\alpha (y - w - \rho)^{1-\alpha}$$

where $\alpha \in (0, 1)$ measures the worker's bargaining power. The wage payoff to the worker is

$$w = \alpha(y - \rho) \quad (4.9)$$

also $\partial w / \partial \rho = -\alpha$. The banker, having anticipated the above negotiated outcome, solves the following

$$\rho = \arg \max \left(\rho - \frac{\gamma}{q(\theta)} \right)^\beta (y - w - \rho)^{1-\beta}$$

and we get

$$\rho = \beta_\alpha(y - w) + (1 - \beta_\alpha) \frac{\gamma}{q(\theta)} \quad (4.10)$$

where $\beta_\alpha = \beta / [1 - \alpha(1 - \beta)]$ is the "effective" bargaining power of the banker, and ρ with a similar interpretation to (4.3). Combining equation (4.9) and (4.10), we get the payoff to the worker, banker, and entrepreneur, respectively

$$w = \alpha(1 - \beta) \left(y - \frac{\gamma}{q(\theta)} \right) \quad (4.11)$$

$$\rho = \beta \left(y - \frac{\gamma}{q(\theta)} \right) \quad (4.12)$$

$$y - w - \rho = (1 - \alpha)(1 - \beta) \left(y - \frac{\gamma}{q(\theta)} \right) \quad (4.13)$$

In contrast to the previous case, equation (4.11) implies that worker's realized share is reduced from α to $\alpha(1 - \beta)$, of $(y - \gamma/q(\theta))$ instead of y . In other words, the sequential bargaining structure endows the worker with a lower effective bargaining power and forces her to share the vacancy burden. As a result, the entry decision to the labor market is now given by

$$\begin{aligned} \alpha(1 - \beta) \left(y - \frac{\gamma}{q(\theta)} \right) \cdot \theta q(\theta) &\geq z \\ \Rightarrow y \geq y^* &= \frac{\gamma}{q(\theta)} + \frac{z}{\alpha(1 - \beta)\theta q(\theta)} \equiv h(\theta) \end{aligned}$$

which suggests that the relationship between the cut-off value y^* and the labor market tightness θ be no longer monotone. We thus have the following

Lemma 4.3.2.1. *Both the cut-off productivity y^* and the average market productivity \hat{y} , are increasing in labor market tightness θ , if and only if*

$$\theta > \theta_1 = \frac{z \left(\frac{1}{\eta} - 1 \right)}{\gamma \alpha (1 - \beta)}$$

where $\eta = -\theta q'(\theta)/q(\theta)$ is the matching elasticity of the worker. When $\theta < \theta_1$, both y^* and \hat{y} are decreasing in θ .

Proof. Check $h'(\theta)$ and use Lemma 4.3.1.1. □

This lemma warrants some discussion. Because wage is decreasing in labor market tightness (see equation (4.11)), as θ increases, even though it becomes more easier for workers to find jobs on the market (since $d[\theta \cdot q(\theta)]/d\theta > 0$), they also have to share a large amount of vacancy burden¹⁰, i.e., $\gamma/q(\theta)$. Thus, y^* (and so does \hat{y}) initially falls with θ but later on only highly productive workers are willing to enter when labor market becomes sufficiently tight. The related threshold, θ_1 , increases with the payoff from home production z and the banker's bargaining power β , and decreases with vacancy cost γ , worker's bargaining power α , and its matching elasticity η . For

¹⁰Note that $1/q(\theta)$ in the duration for an entrepreneur to find a worker. Since $q'(\theta) < 0$, the effective vacancy cost, $\gamma/q(\theta)$, increases as the labor market becomes more tight.

example, a higher γ would induce a wider range of θ in which y^* is increasing since only more skilled workers can afford to share the vacancy cost.

Free entry conditions for bankers and entrepreneurs are, respectively

$$\frac{k}{\phi p(\phi)q(\theta)} = \beta \left(\hat{y} - \frac{\gamma}{q(\theta)} \right) \quad (4.14)$$

$$\frac{c}{p(\phi)q(\theta)} = (1 - \alpha)(1 - \beta) \left(\hat{y} - \frac{\gamma}{q(\theta)} \right) \quad (4.15)$$

the characterization of efficiency is thus given by, from equation (4.6) evaluated at Hosios rule

$$\frac{dW}{dE} \Big|_{\theta=\theta^H} = (1 - \alpha)\theta q(\theta)\hat{y}f(y^*)h'(\theta)\frac{d\theta}{dE} - \alpha \left[\beta\hat{y} + (1 - \beta)\frac{\gamma}{q(\theta)} \right] \cdot q(\theta)p(\phi)$$

if $\theta^H < \theta_1$, $h'(\cdot) \leq 0$ and $d\theta/dE > 0$, $dW/dE < 0$; if $\theta^H < \theta_1$ but $h'(\cdot) > \frac{1-F(y^*)}{\theta f(y^*)}$, $d\theta/dE < 0$, $dW/dE < 0$ as well. Only when $\theta^H > \theta_1$ and $0 < h'(\cdot) < \frac{1-F(y^*)}{\theta f(y^*)}$ it is possible that $dW/dE = 0$, in which $\theta_1 < \theta^H < \bar{\theta}$.

Example 4.3.2.1. *In this example, we use the parameters' values from Table 4.1 except that now $z = 0.2$ and $\beta = 0.01$. The labor market tightness are, $\theta_1 = 0.3904$, $\theta^H = 0.8667$, and $\bar{\theta} = 1.8330$, respectively. The social welfare are, $W(\bar{\theta}) = 0.2909$ while $W(\theta^H) = 0.4797$ and $\frac{dW}{dE} \approx 0$. Again, we have shown that a frictional credit market is preferred to the one with perfect smooth credit, and moreover, it also helps restore the efficiency at Hosios rule (at least locally).*

We now have the following proposition

Proposition 4.3.2.1. *When labor market wage contract is determined after the loan contract in credit market, workers effectively share the vacancy cost since banker now enjoys the first-mover advantage. A frictional credit market can be welfare improving and is thus preferred to a frictionless one. In particular, constrained efficiency can be restored at Hosios rule (at least locally) if the labor market is sufficiently but not too much tight.*

Proof. See above. □

Some readers might be wondering why in this case the constrained efficiency can be restored at Hosios rule. This is because the workers decision making process of entry is altered in the current search sequence, and effectively, they have to share the vacancy cost now. In the first case, labor search goes before credit market search (see fn.3) and a tighter labor market is always preferred from the perspective of all workers. Thus, Hosios condition fails to accommodate those extra effects other than the search externalities. In case II, however, the expected wage payoff is no longer monotonically increasing in the labor market tightness since the search sequence is reversed, i.e. credit search comes before the labor market search. A tighter market makes it easier for a worker to find a job, which nevertheless increases her sharing burden of the vacancy cost at the same time. As a result, only highly productive workers would choose to engage in market production. It is through the individual workers internalization upon entry of her *output externality* that helps the economy regain the constrained efficiency at Hosios rule. In other words, search sequence matters for efficiency.

4.4 Conclusion

In this paper we have shown that credit search and matching frictions can lower equilibrium labor market tightness, and thus might improve social welfare and help restore efficiency at Hosios rule. In particular, when the bargaining structure is built in the way so that workers need to share the vacancy cost, it helps the economy achieve the constrained efficiency at Hosios condition. Otherwise, the workers output externality cannot be internalized and the equilibrium labor market is too tight. In addition, a perfect smooth credit market is never desirable since it would induce too many vacancies posted in equilibrium. Credit market freeze would lead to a complete shutdown of the labor market and reduce the social welfare as well. The study of efficiency characterization in a dynamic setup with endogenous job destruction is left for future research.

4.5 References

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4.6 Appendix

In this Appendix, we revisit ANVs basic environment and show that, if the government can charge the total vacancy fee (or equivalently, charge a tax or license fee in addition to the market vacancy cost), the resulted labor market tightness is too low at Hosios rule. Since now entrepreneurs vacancy cost is a source of government tax revenue, they cancel each other in the aggregate income, and the social welfare is given by

$$W = zF(y^*) + m(\theta) \int_{y^*}^1 yf(y)dy$$

where $m(\theta)$ is the worker's probability of getting employed in the labor market, and $y^* = z\alpha \cdot m(\theta)$ is the cut-off productivity. The free-entry condition for the entrepreneur is

$$-\gamma + \frac{m(\theta)}{\theta}(1 - \alpha) \int_{y^*}^1 \frac{yf(y)}{1 - F(y^*)} dy = 0$$

The derivative of W with respect to v , evaluated at Hosios conditon, is

$$\frac{dW}{dv} \Big|_{1-\alpha=\frac{\theta m'(\theta)}{m(\theta)}} = \frac{d\theta}{dv} \cdot \left\{ [(1 - \alpha)y^*]^2 \frac{m(\theta)}{\theta} f(y^*) + \gamma(1 - F(y^*)) \right\} > 0$$

that is, less than the optimal amount of vacancies are posted in equilibrium. In other words, the vacancy fee charged by the government is too high and a deviation from Hosios rule might be optimal.

CHAPTER 5. FUTURE WORK SUMMARY AND DISCUSSION

One way to address insufficient fluctuations in the labor market in Chapter 2 is to extend the search and matching model into a dynamic setup in which firms live forever and the job-worker pair operates for a given period of time that is pinned down by an explicit probability of separation. In this sense, we will need to solve an occasionally binding vacancy constraint as in Petrosky-Nadeau et al. (2018). Also, the model may generate multiple labor market equilibria if both the level and duration of the unemployment benefit become part of the policy instruments and a selection criterion is thus needed.

For Chapter 3, it is of great interest to calculate the welfare cost of a frictional interbanking market as well as the "savings glut externality" as emphasized in Boissay et al. (2016). In addition, it would be more interesting to both qualitatively and quantitatively characterize any macro-prudential or time-consistent government policies that explicitly address the inefficiencies and/or externalities. An extended evaluation of government programs such as pre-crisis saving tax or post-crisis financial bailout would help us better understand the interactions between the financial frictions and real economy.

As mentioned in Chapter 4, once properly extended to a dynamic search and matching framework, the role of a frictional credit market in terms of deterring pre-crisis less-efficient vacancy creation can then be examined quantitatively. In this sense, it may complement the recent work of Petrosky-Nadeau and Wasmer (2013) in which the existence of a frictional credit market helps destroy those inefficient jobs in the post-crisis era and speed up the recovery of the economy.

5.1 References

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